Precalculus Student-Directed Review/Enrichment

The following activities will give you an opportunity to apply trigonometry and functions to new situations.

If you need extra support in any of these topics, log into Mathspace (https://bit.ly/fcpsmathspace) using your regular FCPS username and password, and navigate to the associated topic in the eBook. You will find explanations and videos there.

Contents of this Packet:

Biorhythms Project – use transformations of sine curves to predict when you will have good days and bad days!

Average Temperatures – use trigonometry to examine temperature variations in different parts of the world.

Functions Selfie Project – asks you to seek out representations of functions in your world, and learn how to use Desmos to represent those functions.

# Sine Curve and Biorhythms

Biorhythms are 3 cycles -- **physical, emotional, and intellectual** -- thought to affect our behavior and performance. Web sites on biorhythms cite famous people -- Madonna, Johnny Depp, Alicia Keys -- and allude to biorhythmic forces at universally known momentous times in their lives. Marilyn Monroe took a fatal dose of medication on one of her critical days while Sirhan Sirhan shot Bobby Kennedy on one of his critical days.

One author claims that people take risks on critical days that they would not take on other days. He says that the Canadian Royal Air Force surveyed a number of its soldiers' heroic actions. Eighty-eight percent occurred on critical days. Other statistics indicate biorhythms are a significant predictor of good and bad performances.

The Theory of Biorhythms states that there are 3 "cycles" to your life, which started on the day you were born:
**The Physical Cycle: 23 days long**
**The Emotional Cycle: 28 days long**
**The Intellectual Cycle: 33 days long**

Every 23 days the physical cycle (strength, energy, endurance, and resistance to disease) completes a full swing from neutral to high to neutral to low and back to neutral, **resulting in a pattern that graphically resembles the sine function from 0 to $2\pi$.** The days between neutral and high, high and neutral, neutral and low, and low and neutral are not quantifiable except to say that they are above neutral or below neutral.

Every 28 days, you complete an emotional cycle that includes periods of elation, sadness, moodiness, and creativity. On high, peak days you are most likely to be elated and creative, and on low days you are the opposite.

The intellectual rhythm (alertness, memory, and reasoning ability) completes a full cycle every 33 days. On peak days you think your clearest.

We draw the biorhythm curves similar to sine curves. At the high points in each cycle you are at your peak physically, emotionally, or intellectually. These are the days when athletic records are set, you are feeling on top of the world, or you seem particularly smart. These peaks occur approximately once a month for each cycle. The low points are not considered your bad days – not days when you should stay in bed -- but rather days when your mind and body are at rest. It is the neutral times, called "critical days," when you should be careful lest bad things befall you. The critical days are those when the curve crosses the axis of your graph. Beware!

Now that we are ready to find out if you should enter into a new relationship, try to go for a record in the mile, or warn your friends that you are not to be crossed today, we will learn how to calculate your biorhythm.

# Sine Curve and Biorhythms

Instructions to graph your Biorhythms:

On the day of your birth all three cycles began at the same time. To find out where you are in your three biorhythm cycles we need to find out **<u>how many days that you have been alive</u>**. Consider the following example:

For example, suppose John was born on December 20, 2001, and it is now March 1, 2020. He must first figure the number of days he has lived from his birth date to his last birthday – December 20, 2019.

1) Multiply 365 by his age, 18 in this -- example 365 X 21 = 7665.
2) Then add an extra day for each leap year ('92, '96, '00, '04, and '08). There are 5 in this example.
3) Since it is now November 1, 2012, he must add the number of days since his last birthday, in this case: 31(Dec 20-Jan 20) + 31(Jan 20-Feb 20) + 29!(Feb 20-Mar 20) + 31(Mar 20-Apr 20) + 30 (Apr 20-May 20) + 31 (May 20-June 20) + 30 (June 20-July 20) + 31(July 20-Aug 20) + 31 (Aug 20-Sept 20) +30 (Sept 20-Oct 20) +12 (from Oct 20 to Nov. 1).
4) Thus, the number of days he has been alive is 7665 +5 + 317 = 7987

Now you figure out the number of days that you have been alive by March 1, 2020.

Birthdate:______________ #days alive = ________________

Show work using above 4 steps.

Now we are ready to find our biorhythm graphs. To use the graphing calculator remember the following:

a. Make sure that you are in radians.

b. We need to graph 3 sine curves for the three different bio cycles. To do this we need to decide on amplitude for all three graphs. Let's agree on '3'. (FYI Any amplitude will be fine as long as they are all the same.) So each graph will be

   y = 3 sin Bx <u>where x is the number of days that you have been alive</u>.

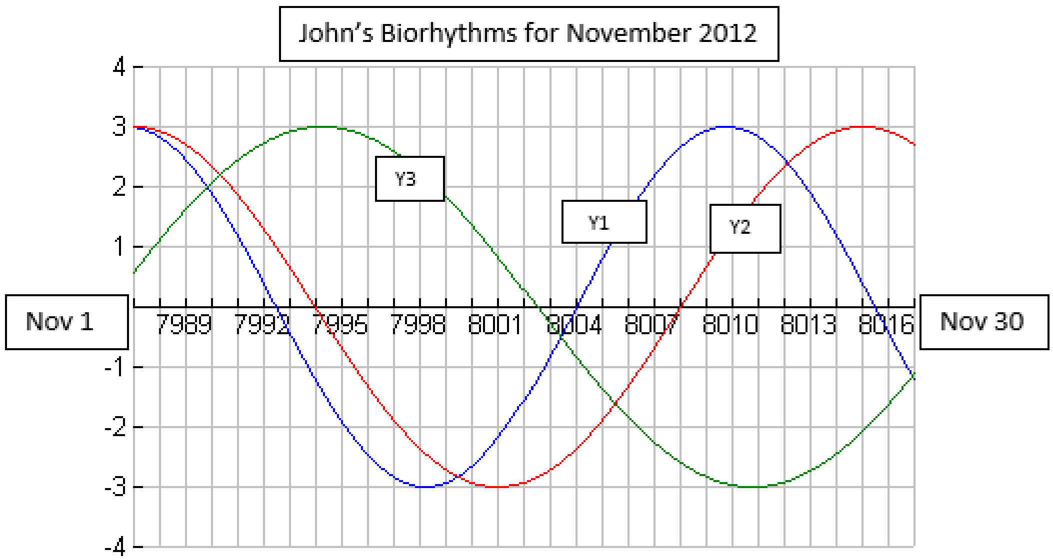
c. All three graphs have a different period so we need to find 3 B values if we remember that period = $2\,\boldsymbol{\pi/B}$
   Physical has period of 23 days so $B_p$ = __________ so $y_1$= 3 sin ___x
   Emotional has period of 28 days so $B_e$ = __________ so $y_2$ = 3 sin ___x
   Intellectual has period of 33 days so $B_i$ = __________ so $y_3$ = 3 sin ___x

Now, before you look on next page think of what your window should look like to see your Biorhythms for the month of March 2020.

xmin = ________ xmax = ________ (xscl = 1) ymin = ________ y max = _________ (yscl = 1)

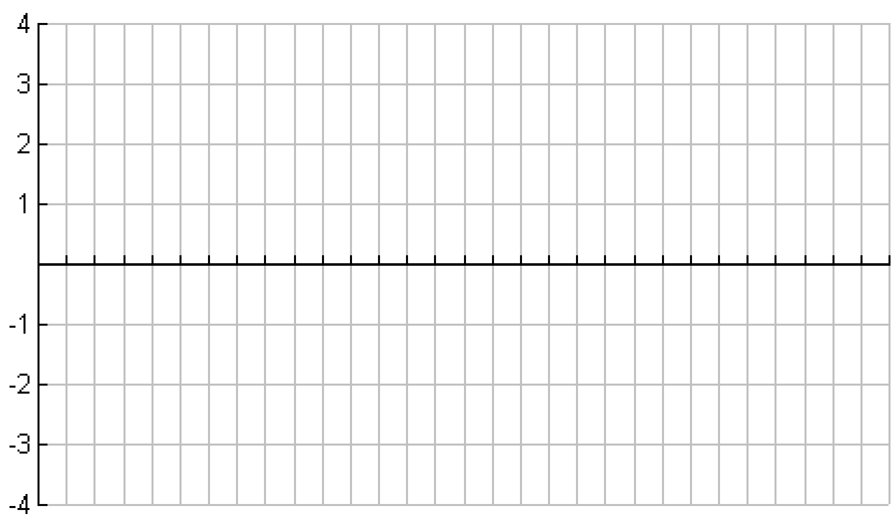
# Sine Curve and Biorhythms

Note in our example for John born on Dec. 20, 1990 he had lived 7987 days as of Nov 1, 2012. So to see month of Nov 2012 the domain should be [7987, 8017] (xmin and xmax) over 30 days. The range should be around [-4, 4] (ymin and ymax) to see max and min values.




Identify John's good, off, and critical days for November. ________________________________

Now find your graphs for March 2020. Put these graphs in your calculator:
$y_1(x)$:=3sin(2 $\pi$ /23 x) Physical
$y_2(x)$:=3sin(2 $\pi$ /28 x) Emotional
$y_3(x)$:=3sin(2 $\pi$ /33 x) Intellectual
The graphing calculator/Desmos makes this very easy! <u>Sketch</u> them on the graph below. Make sure to label which graphs are which and your x axis dates. Use your xmin and xmax for your birthdate.




Identify <u>your</u> high, off, and critical days for March: _______________________

Name: ________________________________

Period: _________ Date: ________

**Monthly Average Temperatures**

The monthly average temperatures (in °F) in Vancouver, Canada, are shown in the table.

| Month | Temp (°F) | | Month | Temp (°F) |
|---|---|---|---|---|
| Jan | 36 | | July | 64 |
| Feb | 39 | | Aug | 63 |
| Mar | 43 | | Sept | 57 |
| Apr | 48 | | Oct | 50 |
| May | 55 | | Nov | 43 |
| June | 59 | | Dec | 39 |

a) Plot the monthly average temperatures over a **2-year period** by letting x = 1 correspond to the month of January during the first year. *Sketch* your graph here. Do the data seem to indicate a particular pattern?

b) Graph the line y = 50 on the same grid. What does this line represent with regard to temperature in Vancouver?

c) Approximate the amplitude, period, and phase shift of the translated sine wave indicated by the data.

d) Determine a sinusoidal function of the form $f(x) = a\sin(bx + c) + d$ , where a, b, c, and d are constants, that models the data.

e) Graph f together with the data on the same coordinate axes. How well does f model the given data? Compare your equation and graph with a partner. How similar/different were your equations?

f) Use the sine regression capability of a graphing calculator to find the equation of a sine curve that fits these data.

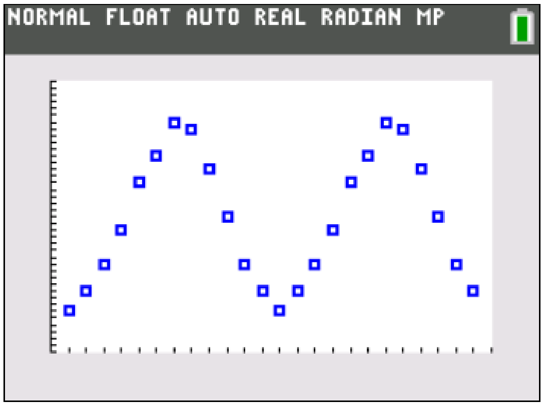
g) Compare the values of your estimated model versus the calculated model. Write a few sentences about the similarities and differences in the two models.

h) Find two other equations (a sine and a cosine) for the data. Compare your equations with a partner.

i) Search online for temperature data for a city in the southern hemisphere, and repeat steps a-f. What do you notice?

### Solution

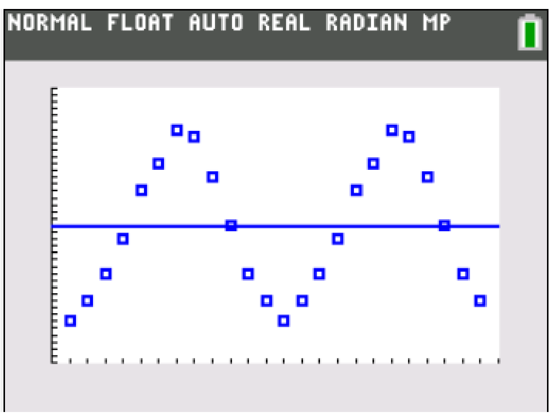
a) yes




b) The average of the maximum and minimum temperatures is a good choice for d.

$$d = \frac{36+64}{2} = 50$$

Sine function oscillates around y = 0, but this graph is a vertical translation with 50 units up. Line y=50 represents “middle” points of the wave.




c) We can use the maximum and minimum monthly average temperatures to find the amplitude a.

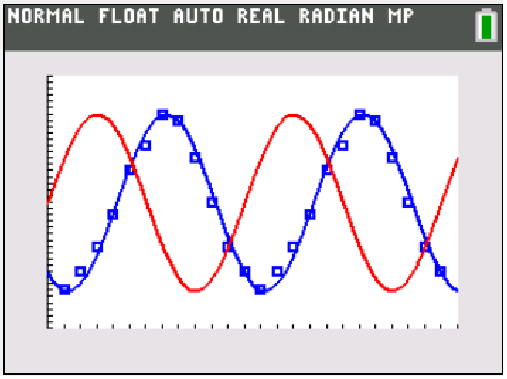
$$a = \frac{64-36}{2} = 14$$

Since temperatures repeat every 12 months, period is T = 12.
The coldest month is January (x = 1), and the hottest month is July (x = 7), so we should choose phase shift to be about 4. The table shows that temperatures are actually a little warmer after July than before, so we experiment with values just greater than 4 to find phase shift. Trial and error with a calculator leads to phase shift = 4.2

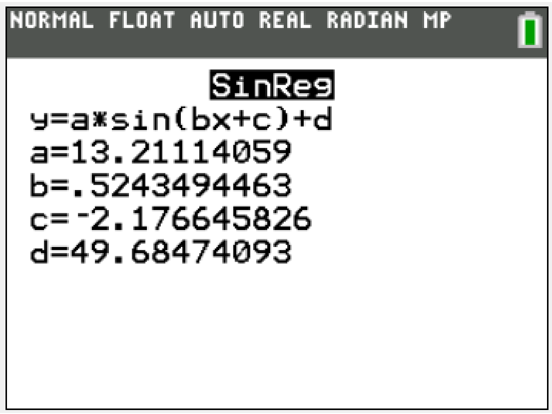
d) Since T = 12, b is $\frac{2\pi}{12} = \frac{\pi}{6}$.

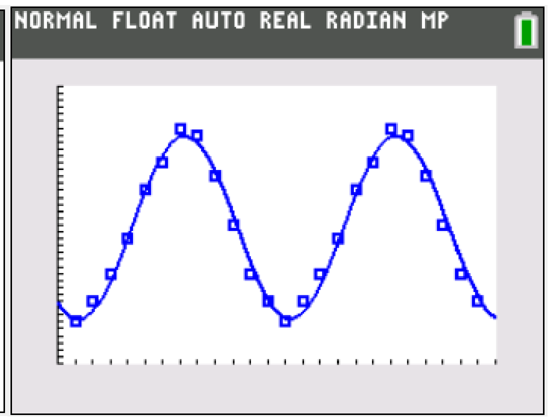
Therefore the estimated model is $f(x) = 14\sin\left[\frac{\pi}{6}(x-4.2)\right] + 50 = 14\sin\left(\frac{\pi}{6}x - \frac{4.2\pi}{6}\right) + 50$

e) The function gives an excellent model for the data. The figure also shows the graph of $f(x) = 14\sin\left(\frac{\pi}{6}x\right) + 50$ for comparison. The horizontal translation of the model is fairly obvious here.




f) Using two years of the given data and the sine regression feature of the graphing calculator, we determine a second model for these data, showed in the first graph, while the next one shows the graph along with the data points.



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The second model is $f(x) = 13.21\sin\left(.52x - 2.18\right) + 49.68$

g) Comparison

| | Estimated model | Calculated model |
|---|---|---|
| a | 14 | 13.21 |
| b | $\frac{\pi}{6} = 0.52$ | 0.52 |
| d | $-\frac{\pi}{6} \cdot 4.2 = 2.199$ | -2.18 |
| c | 50 | 49.68 |

h)

$$f(x) = 13.21\sin(.52x - 2.18 + 2\pi) + 49.68 =$$
$$13.21\sin(.52x + 4.10) + 49.68$$

$$f(x) = 13.21\cos\left(\frac{\pi}{2} - .52x + 2.18\right) + 49.68 =$$
$$13.21\cos(-.52x + 3.75) + 49.68$$

Precalculus | Name___________________________
Performance Task | Period ________ Date__________

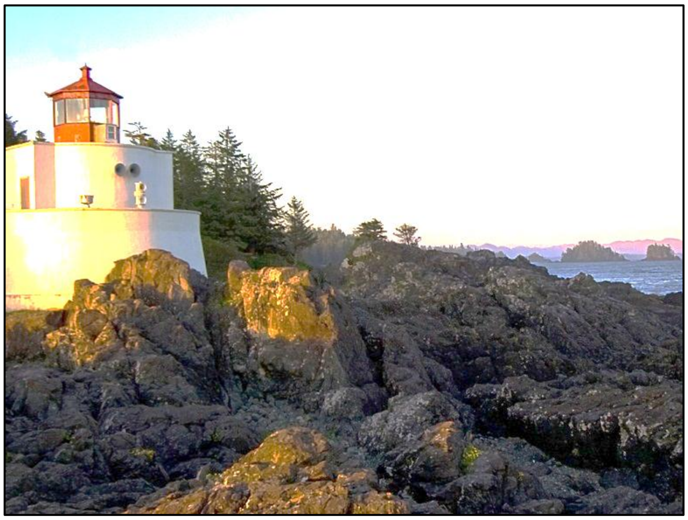
Throughout the year, you have learned about many different types of functions and their characteristics. Now it's time to discover where these types of functions exist outside of the classroom. You will be completing an in class project that collects, analyzes and presents different real-world examples of these functions. Here are the details of the project:

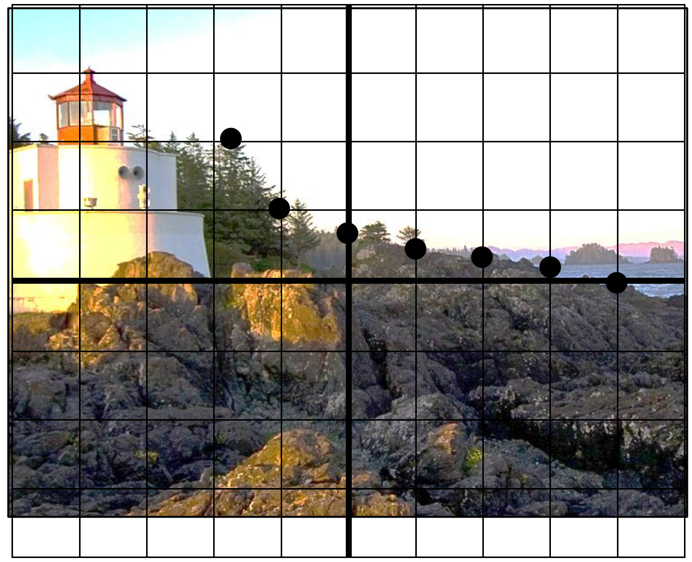
1. You will use at least one selfie that you have taken. Your photo should include yourself and a feature that can be traced like a function, for example the shape of a structure, a skyline, an object in the photo, etc. Each photo must consist of the following:

2. Your photo(s) must represent at least one of the following functions:
   a. Sine/Cosine
   b. Tangent
   c. Polynomial with Degree $\geq$ 2
   d. Exponential
   e. Logarithmic
   f. Rational
   g. Piecewise

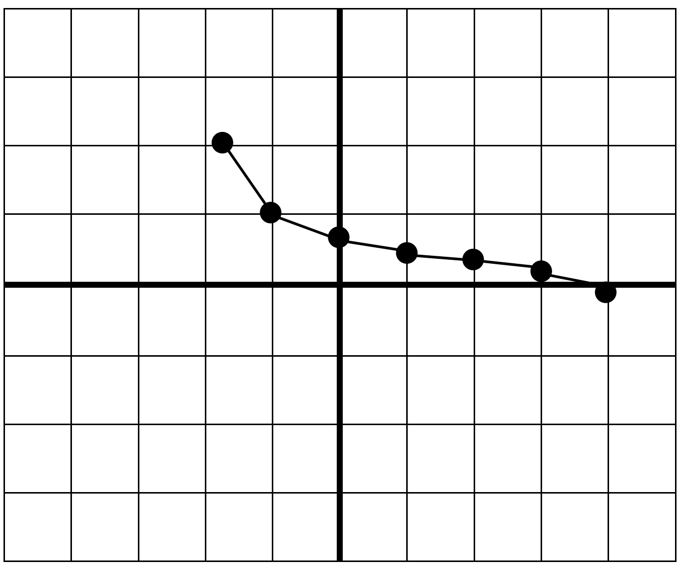
3. See directions on the last page on how to upload your image into Desmos and create your curves. You can save and share your Desmos file with your friends and your teacher.

The photograph below has been placed on a coordinate plane. Then, points were plotted to identify the graph.





Based on the points plotted, the graph appears to represent the function ____________________.



The graph appears to have the following transformations:

- 
- 
- 

Therefore, the equation with the transformations would be:

The domain is:

The range is:

The x-intercept appears to be:

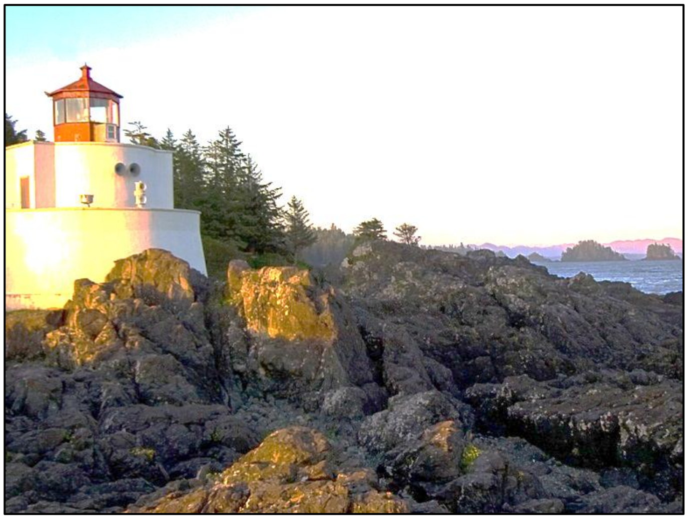
The y-intercept appears to be:

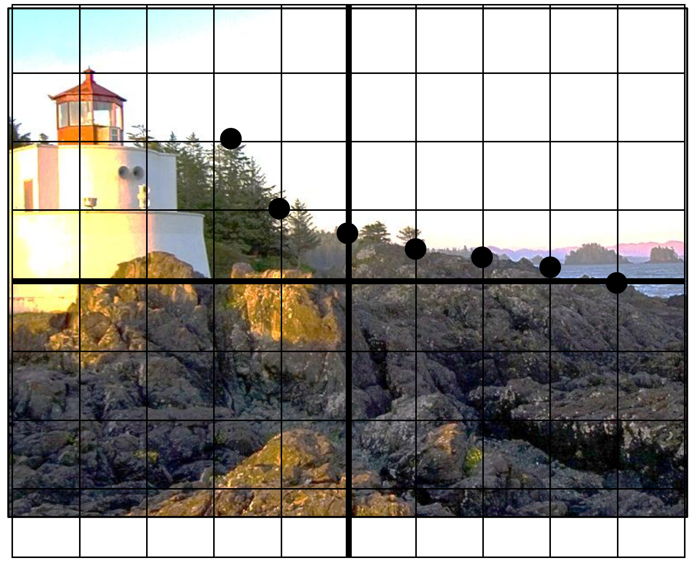
The end behavior is:

How do you think the model would change if you include the tip of the lighthouse as a point?

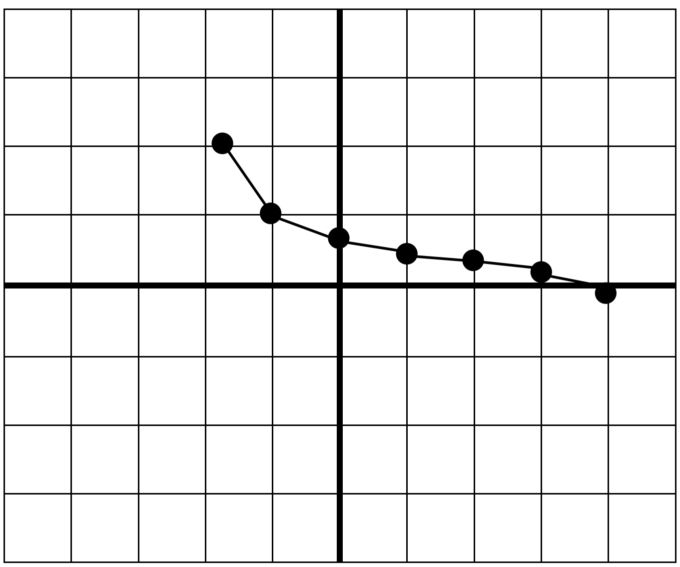
**Answer Key**

The photograph below has been placed on a coordinate plane. Then, points were plotted to identify the graph.





Based on the points plotted, the graph appears to represent the function: __logarithmic_____________.



The graph appears to have the following transformations:

- Reflection over x-axis
- Vertical shift up 1
- Horizontal shift left 2

Therefore, the equation with the transformations would be:

$$f(x) = -\log(x+2) + 1$$

The domain is: $(-2, \infty)$

The range is: $(-\infty, \infty)$

The x-intercept appears to be: $(3.5, 0)$

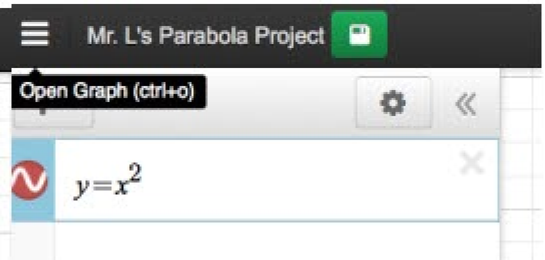
The y-intercept appears to be: $(0, 0.75)$

The end behavior is:

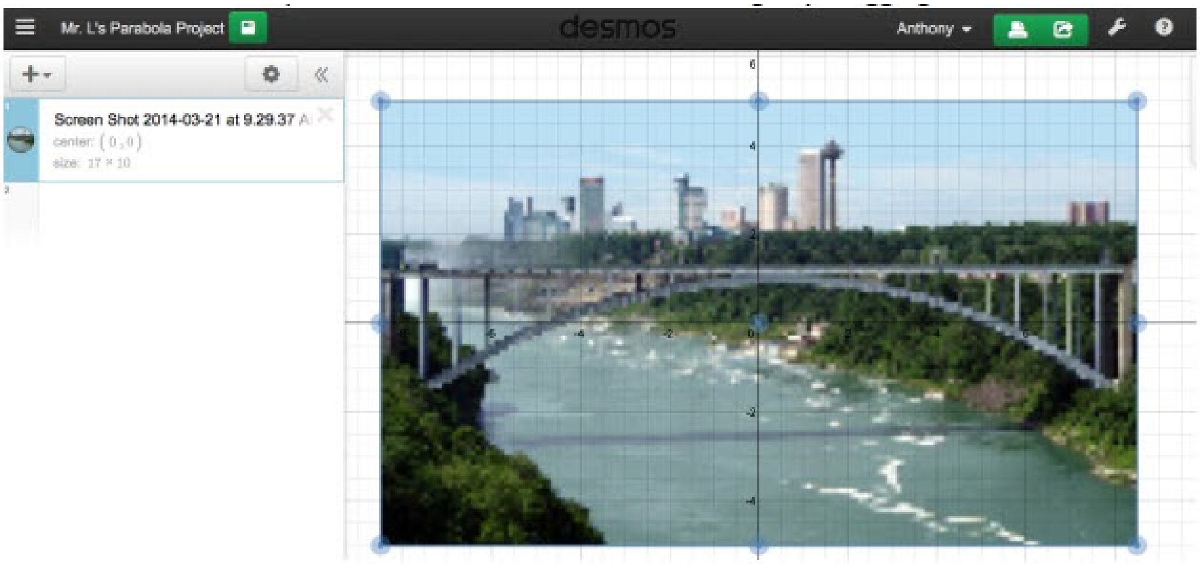
Left: $\lim_{x \to -2} f(x) = \infty$ Right: $\lim_{x \to \infty} f(x) = -\infty$

**Inserting Your Selfie into Desmos:**

Inserting: Go to www.desmos.com and launch the calculator. Log in using Google with your fcpsschools account. Before you start working on your project you are going to want to name it. In the top left corner click "Untitled Graph". Save the graph as your project name. Once you created and name and start working on your project there will be a green save button that will appear. SAVE YOUR GRAPH PERIODICALLY.


Working on your project: Click on the + in the upper left corner and choose image. Navigate to your picture.




Now the work begins. You are going to use your image and overlay functions that correspond to your real-life object. All of your functions will be created and listed on the left-hand side.