## Geometry Student-Directed Review/Enrichment

The following activities are related to topics that you have learned about earlier this year. You may choose to work your way through all of the activities in order, or to prioritize working on activities for topics that you don't remember as well or that you struggled with earlier in the year.

If you need extra support in any of these topics, log into Mathspace (https://bit.ly/fcpsmathspace) using your regular FCPS username and password, and navigate to the associated topic in the eBook. You will find explanations and videos there.

Contents of this Packet:
Unit 1 - Geometry Basics (Distance and Midpoint)
Deriving the Distance Formula
Deriving the Midpoint Formula
Lake Geometria
Task - Baseball Diamond
Unit 2 - Logic and Reasoning
Vocabulary and Symbols
Logic and Conditional Statements (Parts 1 and 2)
Which Law of Logic?
Inductive and Deductive Reasoning (Parts 1 and 2)
Algebraic Properties of Equality
Units 4-7-Triangles
How Many Triangles?
Which Triangles are Similar?
Unit 8 - Polygons (Quadrilaterals)
Properties of Quadrilaterals (Parts 2 and 3)
Zombie Quadrilateral

## Deriving the Distance Formula

Name $\qquad$ Date $\qquad$

1. Use the diagram below to answer the following questions.

a. What is $A C$ ?
b. What are the coordinates of $A$ and $C$ ?
c. Use the coordinates of $A$ and $C$ to compute the length of $\overline{A C}$. Show your work.
d. What are the coordinates of $C$ and $B$ ?
e. Use the coordinates of $C$ and $B$ to compute the length of $\overline{C B}$. Show your work.
f. Draw the segment $\overline{A B}$. What kind of triangle is $\triangle A B C$ ? For that kind of triangle, what are $\overline{A C}$ and $\overline{C B}$ called? What is $\overline{A B}$ called?
g. How can you find the length of the hypotenuse of a right triangle if you know the lengths of the two legs?
h. Use your answer to $g$ to find the length of $\overline{A B}$.
2. Use the diagram below to answer the following questions.

a. On the diagram above, create a right triangle with a horizontal leg, vertical leg, and hypotenuse $\overline{A B}$. Label the vertex of the right triangle $C$. Is this the only right triangle you could have drawn? Explain.
b. Find the length of $\overline{A C}$ and $\overline{C B}$.
c. Use the lengths of $\overline{A C}$ and $\overline{C B}$ to find the length of $\overline{A B}$.
3. The endpoints of a vertical segment $\overline{A B}$ are $\mathrm{A}\left(x_{1}, y_{1}\right)$ and $\mathrm{B}\left(x_{2}, y_{2}\right)$. Use this diagram for the following questions. (Do not count. Graph is not to scale.)

a. Label the lower point $A\left(x_{1}, y_{1}\right)$ and the upper point $\mathrm{B}\left(x_{2}, y_{2}\right)$. Because $\overline{A B}$ is a vertical segment, what can you say about $x_{1}$ and $x_{2}$ ?
b. Express the length of $\overline{A B}$ in terms of $y_{1}$ and $y_{2}$.
c. Express the length of $\overline{A B}$ in a different way in terms of $y_{1}$ and $y_{2}$.
d. Why is it necessary to use absolute value for the formulas above?
e. Does it matter which of the two formulas above you use?
f. Write a formula for the length of $\overline{A B}$ using either formula above.

$$
\overline{A B}=
$$

g. The endpoints of a vertical segment are $G(-10,12)$ and $H(-10,-22)$. Use one of your formulas to compute the length of $\overline{G H}$.
4. The endpoints of a horizontal segment $\overline{C D}$ are $C\left(x_{1}, y_{1}\right)$ and $D\left(x_{2}, y_{2}\right)$. Use this diagram for the following questions. (Do not count. Graph is not to scale.)

a. Label the point on the left $C\left(x_{1}, y_{1}\right)$ and the point on the right $D\left(x_{2}, y_{2}\right)$. Since $\overline{C D}$ is a horizontal segment, what can you say about $y_{1}$ and $y_{2}$ ?
b. Express the length of $\overline{C D}$ in two different ways in terms of $x_{1}$ and $x_{2}$.
c. Write a formula for the length of $\overline{C D}$ using either formula above.

$$
\overline{C D}=
$$

d. The endpoints of a horizontal segment are $E(-10,12)$ and $F(24,12)$. Use your formula to compute the length of $\overline{E F}$.
5. The endpoints of a segment $\overline{P Q}$ are $P\left(x_{1}, y_{1}\right)$ and $D\left(x_{2}, y_{2}\right)$. Use this diagram for the

following questions. (Do not count. Graph is not to scale.)
a. Use the relationship between the legs and hypotenuse of a right triangle found in the Pythagorean theorem to complete the equation below:

$$
(P Q)^{2}=
$$

b. Label the point $P\left(x_{1}, y_{1}\right)$ as and point $Q$ as $\left(x_{2}, y_{2}\right)$. Find the coordinates of the point $R$.
c. Write formulas for the lengths of $\overline{P R}$ and $\overline{Q R}$ using either formula above.

$$
\overline{P R}=\quad \overline{Q R}=
$$

d. Determine whether this equation is true. If it is true, explain why. If it is false, give a counterexample. (Hint: When you square a real number, is it ever negative?)

$$
|a|^{2}=a^{2}
$$

e. Use the last three problems to get a formula for $(\overline{P Q})^{2}$ in terms of $x_{1}, x_{2}, y_{1}$, and $y_{2}$.

$$
(\overline{P Q})^{2}=
$$

Mathematics Instructional Plan - Geometry
f. Take the square root of both sides of your last formula to write $\overline{P Q}$ in terms of $x_{1}, x_{2}, y_{1}$, and $y_{2}$.

$$
\overline{P Q}=
$$

g. The endpoints of a segment are $I(8,12)$ and $J(2,4)$. Use the formula you found to compute the distance between $I$ and $J$.

## Deriving the Midpoint Formula

Name $\qquad$ Date $\qquad$

1. The midpoint of a segment is the point on the segment that is the same distance from both endpoints. Use the graph below to answer the following questions.

a. What are the $x$-coordinates of $A$ and $C$ ? $\qquad$ and $\qquad$
b. What is the average (or mean) of the $x$-coordinates of $A$ and $C$ ? $\qquad$
c. What number is halfway between the $x$-coordinates of $A$ and $C$ ? $\qquad$
d. What are the coordinates of the midpoint of $\overline{A C}$ ? $\qquad$ Graph and label the midpoint.
e. Explain the relationships among the answers to questions $b, c$, and $d$.
f. The endpoints of a vertical segment are $G(-10,12)$ and $H(-10,-22)$. Use your formula to compute the midpoint of $\overline{E F}$.
2. Use the graph below to answer the following questions.

a. What are the $y$-coordinates of $B$ and $C$ ? $\qquad$ and $\qquad$
b. What is the average (or mean) of the $y$-coordinates of $B$ and $C$ ? $\qquad$
c. What number is halfway between the $y$-coordinates of $B$ and $C$ ? $\qquad$
d. What are the coordinates of the midpoint of $\overline{B C}$ ? $\qquad$ Graph and label the midpoint.
e. Explain the relationships among the answers to questions $b, c$, and $d$.
f. $B(0,-2)$ is the midpoint of segment $\overline{C D}$. If point C is located at $(0,4)$ as in the figure above, what would be the coordinate of point $D$ ?
3. Use the diagram below to answer the following questions.

a. Graph the midpoint of $\overline{A C}$. Label it $P$. What is the $x$-coordinate of this point? $\qquad$
b. What is the average (or mean) of the $x$-coordinates of $A$ and $C$ ? $\qquad$ How is this related to your answer to a?
c. Graph the midpoint of $\overline{B C}$. Label it Q . What is the y -coordinate of this point? $\qquad$
d. What is the average (or mean) of the $y$-coordinates of $B$ and $C$ ? $\qquad$ How is this related to your answer to c?
e. What is the average (or mean) of the $x$-coordinates of $A$ and $B$ ? $\qquad$
f. What is the average (or mean) of the $y$-coordinates of $A$ and $B$ ? $\qquad$
g. What is the midpoint of $\overline{B C}$ ?
h. How is the midpoint of $\overline{A B}$ related to the answers to 3 e and 3 f ?
4. Use the diagram below to answer the following questions.

a. What is the average (or mean) of the $x$-coordinates of $A$ and $B$ ? $\qquad$
b. What is the average (or mean) of the $y$-coordinates of $A$ and $B$ ? $\qquad$
c. What are the coordinates of the midpoint of $\overline{A B}$ ? $\qquad$
d. Explain the relationships among the answers to questions $b, c$, and $d$.
5. The endpoints of a segment $\overline{P Q}$ are $P\left(x_{1}, y_{1}\right)$ and $Q\left(x_{2}, y_{2}\right)$.

a. Label the point $P$ as $\left(x_{1}, y_{1}\right)$ and point $Q$ as $\left(x_{2}, y_{2}\right)$.
b. Write a formula for the average (or mean) of the $x$-coordinates of $P$ and $Q$.
c. Write a formula for the average (or mean) of the $y$-coordinates of $P$ and $Q$.
d. One way to think of the midpoint of $\overline{P Q}$ is as follows: average (or mean) of the $x$ coordinates, average (or mean) of the $y$-coordinates. Use this to derive a formula for the midpoint of $\overline{P Q}$.
e. The endpoints of a segment are $E(8,12)$ and $F(2,4)$. Use your formula to compute the midpoint of $\overline{E F}$.

## Lake Geometria

## Name

$\qquad$ Date $\qquad$
The islands of Lake Geometria are shown below. A cabin is marked on each island. The scale, using units called stades, is shown in the lower right. A stade measures about 600 feet, so Lake Geometria is not very big. Use the grid to help you answer the following questions. All island

measurements should be made from cabin to cabin.

1. What is the distance in stades from Eudoxus to Archimedes? (Round to the nearest stade.) Describe how you found your answer.
2. What is the distance in stades from Thales to Euclid? (Round to the nearest stade.) Describe how you found your answer.

Mathematics Instructional Plan - Geometry
$\qquad$

## Take Me Out to the Ball Game

The four bases of a major league baseball field form a square which is 90 feet on each side. A drawing of the field is overlaid on a coordinate grid.

- The pitching mound is collinear to home plate and second base.
- The pitching mound is not equidistant from each base.
- The pitching mound is 60.5 feet from home plate.


To which base is the pitcher closest?
Mathematically justify your answer and provide a labeled diagram which models the problem and shows all variables to which you will refer.

## Vocabulary and Symbols

Name $\qquad$ Date $\qquad$
Define each of the following vocabulary terms.

| Term | Definition |
| :---: | :---: |
| Conditional <br> Statement | As__ that can be w___ in "if ..., then" form. |
| Hypothesis | The part of a c____ statement that f ___ "if." |
| Conclusion | The part of a conditional s___ that follows the word "then." |
| Negation | The o $\qquad$ of a given s $\qquad$ formed by adding or removing the word n $\qquad$ from the statement. |
| Negate | To add or remove the word n $\qquad$ from a statement to change its truth value from true to $f$ $\qquad$ or from false to $t$ $\qquad$ . |
| Converse | As $\qquad$ formed from a c $\qquad$ statement by s $\qquad$ the h $\qquad$ and the c $\qquad$ |
| Inverse | As $\qquad$ formed from a c $\qquad$ statement by n $\qquad$ the $h$ $\qquad$ and the C $\qquad$ |
| Contrapositive | As $\qquad$ formed from a c $\qquad$ statement by S $\qquad$ AND n $\qquad$ the <br> h $\qquad$ and the c $\qquad$ . |
| Biconditional | A statement that combines the c $\qquad$ and its C $\qquad$ when they are both true. It uses the phrase "if and o $\qquad$ if." |

Fill in the meaning of each of the following symbols.


## Vocabulary and Symbols (Teacher's Reference)

Define each of the following vocabulary terms.

| Term | Definition |
| :--- | :--- |
| Conditional <br> Statement | $\underline{\text { A statement that can be written in "if ..., then" form. }}$ |
| Hypothesis | The part of a conditional statement that follows the word "if." |
| Conclusion | The part of a conditional statement that follows the word "then." |
| Negation | The opposite of a given statement formed by adding or removing the word not from <br> the statement. |
| Negate | To add or remove the word not from a statement to change its truth value from true <br> to false or from false to true. |
| Converse | A statement formed from a conditional statement by switching the hypothesis and <br> the conclusion. |
| Inverse | A statement formed from a conditional statement by negating the hypothesis and <br> the conclusion. |
| Contrapositive | A statement formed from a conditional statement by switching AND negating the <br> hypothesis and the conclusion. |
| Aiconditional | A statement that combines the conditional and its converse when they are both true. <br> It uses the phrase "if and only if." |

Fill in the meaning of each of the following symbols.

| $\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \boldsymbol{s}, \boldsymbol{t}$, |
| :---: | :--- | :---: | :--- |
| etc. | Meaning: Symbols used to represent statements such as hypotheses and conclusions

## Logic and Conditional Statements, Part 1

## Name

$\qquad$ Date $\qquad$
Use the following conditional statement to complete 1-11: "If elephants fly, then fish don't swim." Each answer should be a complete sentence, not symbols.

1. $p$ is the hypothesis. Write $p$.
2. $q$ is the conclusion. Write $q$. $\qquad$
3. $\sim p$ means "the negation of $p$." Write $\sim p$.
4. $\sim q$ means "the negation of $q$." Write $\sim q$.
5. (Converse) $q \rightarrow p$ means " $q$ implies $p$ " or "If $q$, then $p$. ." Write $q \rightarrow p$.
6. (inverse) $\sim p \rightarrow \sim q$ means "Not $p$ implies not $q$ " or "If not $p$, then not $q$." Write $\sim p \rightarrow \sim q$.
7. (contrapositive) $\sim q \rightarrow \sim p$ means "Not $q$ implies not $p$ " or "If not $q$, then not $p$." Write $\sim q \rightarrow \sim p$.
8. $p \wedge q$ means " $p$ and $q$." Write $p \wedge q$.
9. $p \vee q$ means " $p$ or $q$." Write $p \vee q$.
10. $\therefore p$ means "therefore $p$." Write $\therefore p$. $\qquad$
11. $p \leftrightarrow q$ means " $p$ if and only if $q$." Write $p \leftrightarrow q$. $\qquad$

Use the following conditional statement to complete 1-8: "If I win, then you don't lose."

1. Write the hypothesis. $\qquad$
2. Write the conclusion.
3. Negate the hypothesis. $\qquad$
4. Negate the conclusion. $\qquad$
5. Write the converse. $\qquad$
6. Write the inverse. $\qquad$
7. Write the contrapositive. $\qquad$
8. Write the biconditional. $\qquad$

## Logic and Conditional Statements, Part 1 (Teacher's Reference)

Use the following conditional statement to answer1-11: "If elephants fly, then fish don't swim." Each answer should be a complete sentence, not symbols.

1. $p$ is the hypothesis. Write $p$. Elephants fly.
2. $q$ is the conclusion. Write $q$. Fish don't swim.
3. $\sim p$ means "the negation of $p$. " Write $\sim p$. Elephant don't fly.
4. $\sim q$ means "the negation of $q$." Write $\sim q$. Fish swim.
5. (converse) $q \rightarrow p$ means " $q$ implies $p$ " or "If $q$, then $p$." Write $q \rightarrow p$. If fish don't swim, then elephants fly.
6. (inverse) $\sim p \rightarrow \sim q$ means "Not $p$ implies not $q$ " or "If not $p$, then not $q$." Write $\sim p \rightarrow \sim q$. If elephants don't fly, then fish swim.
7. (contrapositive) $\sim q \rightarrow \sim p$ means "Not $q$ implies not $p$ " or "If not $q$, then not $p$." Write $\sim q \rightarrow \sim p$. If fish swim, then elephants don't fly.
8. $p \wedge q$ means " $p$ and $q$." Write $p \wedge q$. Elephants fly and fish don't swim.
9. $p \vee q$ means " $p$ or $q$." Write $p \vee q$. Elephants fly or fish don't swim.
10. $\therefore p$ means "therefore $p$. " Write $\therefore p$. Therefore, elephants fly.
11. $p \leftrightarrow q$ means " $p$ if and only if $q$." Write $p \leftrightarrow q$. Elephants fly, if and only if fish don't swim.

Use the following conditional statement to answer 1-8: "If I win, then you don't lose."

1. Write the hypothesis. I win.
2. Write the conclusion. Idon't lose.
3. Negate the hypothesis. Idon't win.
4. Negate the conclusion. Ilose.
5. Write the converse. If I don't lose, then I win.
6. Write the inverse. If I don't win, then I lose.
7. Write the contrapositive. If I lose, then I don't win.
8. Write the biconditional. I win, if and only if, I don't lose.

## Logic and Conditional Statements, Part 2

Name $\qquad$ Date $\qquad$

1. Write each of the following statements as a conditional statement. Then, circle the hypothesis, and underline the conclusion.
a. Mark Twain wrote, "If you tell the truth, you don't have to remember anything."
b. William Camden wrote, "The early bird catches the worm."
c. Helen Keller wrote, "One can never consent to creep when one feels the impulse to soar."
$\qquad$
d. Mahatma Gandhi wrote, "Freedom is not worth having if it does not include the freedom to make mistakes."
$\qquad$
e. Benjamin Franklin wrote, "Early to bed and early to rise makes a man healthy, wealthy, and wise."
$\qquad$
2. Write the converse, inverse, and contrapositive for each of the following conditional statements. Determine whether each is true or false.
a. "If two segments are congruent, then they have the same length."

Converse: $\qquad$
Inverse: $\qquad$
Contrapositive: $\qquad$
True or false: $\qquad$
b. A rectangle has four sides.

Converse: $\qquad$
Inverse: $\qquad$
Contrapositive: $\qquad$
True or false: $\qquad$
3. Write each of the following statements in symbolic notation:

Let $p$ represent: you see lightning Let q represent: you hear thunder
a. If you see lightning, then you hear thunder. $\qquad$
b. If you hear thunder, then you see lightning. $\qquad$
c. If you don't see lightning, then you don't hear thunder. $\qquad$
d. If you don't hear thunder, then you don't see lightning. $\qquad$
4. Write each of the following statements in symbolic notation:

Let $p$ represent: two planes intersect
Let q represent: the intersection is a line
a. If two planes don't intersect, then the intersection is a line. $\qquad$
b. If the intersection is not a line, then two planes do not intersect. $\qquad$

## Logic and Conditional Statements

Conditional

Statement

$p \longrightarrow q$
or $p$ implies $q$


# Which Law of Logic? 

Name
Date $\qquad$
Directions: Determine what conclusion, if any, can be drawn from the following statements. Indicate whether the law of detachment, or law of syllogism was used.

| Statements | Conclusion | Which Law? |
| :--- | :--- | :--- |
| If Point $A$ divides a segment into <br> two congruent segments, Point $A$ <br> is the midpoint of that segment. <br> Point $M$ divides $\overline{C D}$ into two <br> congruent segments. |  |  |
| If a figure has three sides, then it is <br> a triangle. If a figure is a triangle, <br> then it has three angles. |  |  |
| If a quadrilateral is a square, then <br> it is equilateral. A rectangle is not <br> equilateral. |  |  |
| If you mail the payment by noon, <br> then it will arrive by tomorrow. If <br> your payment arrives by <br> tomorrow, then you will not be <br> charged a late fee. |  |  |
| If the geometry students score <br> 100\% on the test, the teacher will <br> dye her hair. The geometry <br> students scored 100\% on the test. |  |  |
| Right angles are congruent. Angle <br> $A$ and Angle B are right angles. |  |  |
| If Jamal misses practice, then he is <br> not allowed to play in tomorrow's <br> game. Jamal missed practice. |  |  |

The law of contrapositive states that a conditional statement is logically equivalent to its contrapositive. Create a statement that is logically equivalent to the following statement:

If an animal is not a mammal, then it is not a bat.

If a polygon is a hexagon, then the sum of its interior angles is 720 degrees.

## Inductive and Deductive Reasoning: Part 1

Name $\qquad$

Deductive reasoning is a method of reasoning that uses logic to draw conclusions based on definitions, postulates, and theorems.


Date $\qquad$

Inductive reasoning works from more specific observations to broader generalizations.


| Example of Deductive Reasoning | Example of Inductive Reasoning |
| :--- | :--- |
| - Tom knows that if he misses the practice the |  |
| day before a game, then he will not be a <br> starting player in the game. | Observation: Mia came to class late this |
| morning. |  |

A. Complete the following conjectures based on the patterns you observe in specific cases:

Step 1: List all odd numbers from 1-20.
Step 2: Make a table that sums the first two numbers, then the second two numbers.

```
1+3=4
    7 + 11 = 18
3+5=8 13+15=28
5+7=12 15+17=32
```

Step 3: Conjecture: The sum of any two odd numbers is $\qquad$ .
B. Complete the following conjectures based on the patterns you observe in specific cases:
Step 1: List all odd numbers from 1-20. Make a table that multiplies the first two numbers,

$$
\begin{array}{lr}
1 \times 3=3 & 7 \times 11=77 \\
3 \times 5=15 & 13 \times 15=195 \\
5 \times 7=35 & 15 \times 17=255
\end{array}
$$ then the second two numbers.

Step 2: Conjecture: The product of any two odd numbers is $\qquad$ .

## Inductive and Deductive Reasoning: Part 2

Name $\qquad$ Date $\qquad$

1. John always listens to his favorite radio station, an oldies station, when he drives his car. Every morning he listens to his radio on the way to work. On Monday, when he turns on his car radio, it is playing country music. Make a list of valid conjectures (predictions) to explain why his radio is playing different music.
2. $\angle \mathrm{M}$ is obtuse. Make a list of conjectures based on that information.
3. Based the table below, Marina concluded that when one of the two addends is negative, the sum is always negative. Write a counterexample for her conjecture.

| Addends | Sum |  |
| ---: | ---: | ---: |
| -8 | -10 | -18 |
| -17 | -5 | -22 |
| 15 | -23 | -8 |
| -26 | 22 | -4 |

4. Construct a conjecture based on patterns you observe below.

Case 1:
Christine notices that her friend, Endia, is absent from school every first Monday of the month. It is the first Monday of the month. What can she deduce?

Case 3:
Whenever Usain Bolt wears his golden shoes, he wins the 400 -meter race. If he wins this race, then he is the fastest man in the world. Usain Bolt wears his golden shoes to the 2010 Olympics. Make a conjecture.

Case 2:
Mrs. Batten notices that whenever Johnny is nervous he twirls his pencil. Johnny is twirling his pencil. What can Mrs. Batten deduce?

Case 4:
At the Wallops Flight Facility in Virginia, a rocket cannot be launched in the rain. It is raining. Make a conjecture.

## Algebraic Properties of Equality

The algebraic properties of equality can be used to solve and justify $5 x-18=3 x+2$, by writing a reason for each step, as shown in the table below.

| Statement | Reason |
| :--- | :--- |
| $5 x-18=3 x+2$ | Given |
| $2 x-18=2$ | Subtraction Property of Equality |
| $2 x=20$ | Addition Property of Equality |
| $x=10$ | Division Property of Equality |

Using a table like the one above, solve each of the following equations, and state a reason for each step, using the properties contained below.

1. $-2(-w+3)=15$
2. $p-1=6$
3. $2 r-7=9$
4. $3(2 t+9)=30$
5. Given $3(4 v-1)-8 v=17$, prove $v=5$.

| Addition Property of Equality | If $a=b$, <br> then $a+c=b+c$ |
| :---: | :---: |
| Subtraction Property of Equality | If $a=b$, <br> then $a-c=b-c$ |
| Multiplication Property of Equality | If $a=b$, <br> then $a c=b c$ |
| Division Property of Equality | If $a=b$ and $c \neq 0$, <br> then $a \div c=b \div c$ |
| Reflexive Property | If $a=a$ <br> then $b=a$ |
| Symmetric Property | If $a=b$ and $b=c$, <br> then $a=c$ |
| Transitive Property | If $a=b$, then $a$ can be substituted for <br> $b$ in any equation or expression. |
| Substitution Property | $a(b+c)=a b+a c$ |
| Distributive Property |  |

Note: $a, b, a$ nd $c$ are real numbers.
3. Which is closer to Thales—Pythagoras or Heron? Describe how you found your answer.
4. Find a point in the water that is the same distance from Archimedes and Eudoxus. Label the point $M$. Describe how you found this point.
5. Now find a point on land that is the same distance from Archimedes and Eudoxus. Label this point $N$.
6. How many points can you find that are the same distance from Archimedes and Eudoxus? Explain.
7. Find a point on land that is the same distance from Thales and Pythagoras. Is your point on the mainland or on an island? Does it have to be? Explain.
8. Groups staying on any of the islands are provided with solar-charged walkie-talkies, but their ranges are only about 1 mile. There are about 9 stades in a mile. With which islands could someone staying at the cabin on Thales expect to be able to communicate? Show your work, or explain how you found your answer.
9. Estimate the shortest distance from Thales to the mainland. (You may use stades, feet, or miles.) Show your work or explain how you found your answer.
10. Estimate how many miles wide Lake Geometria is at its widest point. Explain how you found your answer.
11. If you sailed from Thales to Pythagora, and your friend sailed from Eudoxus to Heron, would you be sailing parallel to each other? How do you know? Show you work.
12. Is the route from Pythagora to Euclid, perpendicular to the route from Pythagora to Plato? How do you know? Show your work.
13. One ship leaves Archimedes and sails to Euclid, while another ship leaves Eudoxus and sails to Thales. Are they sailing parallel to each other? How do you know? Justify your answer.
14. A buoy located at the midpoint between the islands of Eudoxus and Archimedes. Place an ' $X$ ' on the grid where the buoy would be located.
15. A lighthouse is located at the midpoint between the islands of Pythagora and Euclid. Place a small circle on the grid where the lighthouse would be located.

## How Many Triangles? (Part 1)

Name $\qquad$ Date

1. Mark your pipe cleaner or paper strip at 1-inch intervals. Folding only at your marks, try to make triangles with lengths as given in the table, placing them end-to-end. Use a protractor to measure the angles for each triangle. If no triangle can be formed, then write "none" for the measure of each angle. Complete the table.

| Side Lengths | Sketch | Triangle? <br> Yes/No | Measure of each angle in each triangle formed |
| :---: | :---: | :---: | :---: |
| $4 \mathrm{in} ., 4 \mathrm{in} ., 4 \mathrm{in}$. |  | Yes | $60^{\circ}, 60^{\circ}, 60^{\circ}$ |
| $3 \mathrm{in.}$,5 in., 4 in. |  |  |  |
| 2 in., 6 in., 4 in. |  | No | none |
| $1 \mathrm{in} ., 7 \mathrm{in}$. , 4 in. |  |  |  |
| $2 \mathrm{in.}$,5 in., 5 in. |  |  |  |
| $2 \mathrm{in.}$,7 in., 3 in. |  |  |  |
| 3 in., 6 in., 3 in. |  |  |  |

2. Now use your pipe cleaner or strip to form two sides of a triangle with lengths 5 inches and 7 inches.
a. Could the third side of this triangle with side lengths 5 inches and 7 inches measure ...

1 inch? $\qquad$ 2 inches? $\qquad$ 2.1 inches? $\qquad$ 7 inches? $\qquad$ 11 inches? $\qquad$
11.9 inches? $\qquad$

12 inches? $\qquad$ 13 inches? $\qquad$
b. Complete: The third side must be greater than $\qquad$ inches and less than $\qquad$ inches.
3. Draw a scalene triangle. Measure all three angles. Name the vertex of the largest angle $L$, the vertex of the medium angle $M$, and the vertex of the smallest angle $S$. Measure the three sides. Label the longest side $I$, the medium side $m$, and the shortest side $s$. Which side is opposite?
a. $\angle L$ ? $\qquad$ b. $\angle M$ ? $\qquad$ c. $\angle S$ ?

## How Many Triangles?

Name $\qquad$ Date $\qquad$

1. Determine whether the following lengths will form a triangle. Briefly explain each answer.
a. 5 in., 2 in., 8 in.
b. $6 \mathrm{~cm}, 18 \mathrm{~cm}, 15 \mathrm{~cm}$
c. $5 \mathrm{ft} ., 6 \mathrm{ft}$., 9 ft .
d. 7 in., 7 in., 8 in.
e. $1.2 \mathrm{mi} ., 4.0 \mathrm{mi}$, 1.8 mi .
f. $10 \mathrm{~mm}, 10 \mathrm{~mm}, 0.001 \mathrm{~mm}$
2. List the sides and angles of each triangle in order from smallest to largest. How do you know?

3. List the sides and angles of each triangle in order from largest to smallest. How do you know?

4. $\triangle A B C$ has side lengths of 1 inch, $1 \frac{7}{8}$ inches, and $2 \frac{1}{8}$ inches and angle measures of 90 degrees, 28 degrees, and 62 degrees. Which side is opposite each angle?
5. The lengths of two sides of a triangle are given. Determine the range for the possible lengths of the third side. Explain your reasoning or show work to support your answer.
a. $5 \mathrm{in} ., 2 \mathrm{in}$.
b. $1 \mathrm{~cm}, 180 \mathrm{~cm}$
c. 15 ft ., 10 ft .
d. 70 in., 70 in.
e. $3 \mathrm{mi}, 7 \mathrm{mi}$.
f. $\quad 1.3 \mathrm{~mm}, 1.6 \mathrm{~mm}$

## Which Triangles Are Similar?

Name $\qquad$ Date $\qquad$
Answer the questions using the diagram and angle measures shown below.


| $\mathrm{m} \angle \mathrm{BAC}=66.96^{\circ}$ | $\mathrm{m} \angle \mathrm{DEF}=66.96^{\circ}$ | $\mathrm{m} \angle \mathrm{GIH}=66.96^{\circ}$ | $\mathrm{m} \angle \mathrm{JLK}=66.96^{\circ}$ | $\mathrm{m} \angle \mathrm{NMO}=66.96^{\circ}$ | $\mathrm{m} \angle \mathrm{PRQ}=66.96^{\circ}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{m} \angle \mathrm{ACB}=63.31^{\circ}$ | $\mathrm{m} \angle \mathrm{DFE}=63.31^{\circ}$ | $\mathrm{m} \angle \mathrm{HGI}=63.31^{\circ}$ | $\mathrm{m} \angle \mathrm{JKL}=63.31^{\circ}$ | $\mathrm{m} \angle \mathrm{MNO}=63.31^{\circ}$ | $\mathrm{m} \angle \mathrm{PQR}=65.76^{\circ}$ |
| $\mathrm{m} \angle \mathrm{ABC}=49.73^{\circ}$ | $\mathrm{m} \angle \mathrm{EDF}=49.73^{\circ}$ | $\mathrm{m} \angle \mathrm{GH}=49.73^{\circ}$ | $\mathrm{m} \angle \mathrm{KJL}=49.73^{\circ}$ | $\mathrm{m} \angle \mathrm{MON}=49.73^{\circ}$ | $\mathrm{m} \angle \mathrm{QPR}=47.27^{\circ}$ |

1. Is there enough information to determine whether the triangles are congruent? Explain.
2. What can you say about some of the triangles, using the angle measures? The lengths of the sides are given below:

$\omega$
$\infty$
$\infty$
0
3


| $\mathrm{AB}=3.75 \mathrm{~cm}$ | $\mathrm{DE}=3.75 \mathrm{~cm}$ | $\mathrm{GH}=3.94 \mathrm{~cm}$ | $\mathrm{JK}=3.78 \mathrm{~cm}$ | $\mathrm{MN}=3.20 \mathrm{~cm}$ | $\mathrm{PQ}=3.78 \mathrm{~cm}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{AC}=3.20 \mathrm{~cm}$ | $\mathrm{DF}=3.86 \mathrm{~cm}$ | $\mathrm{GI}=3.26 \mathrm{~cm}$ | $\mathrm{JL}=3.67 \mathrm{~cm}$ | $\mathrm{MO}=3.75 \mathrm{~cm}$ | $\mathrm{PR}=3.75 \mathrm{~cm}$ |
| $\mathrm{BC}=3.86 \mathrm{~cm}$ | $\mathrm{EF}=3.20 \mathrm{~cm}$ | $\mathrm{HI}=3.82 \mathrm{~cm}$ | $\mathrm{KL}=3.14 \mathrm{~cm}$ | $\mathrm{NO}=3.86 \mathrm{~cm}$ | $\mathrm{QR}=3.02 \mathrm{~cm}$ |

3. Are any of the triangles congruent to $\triangle A B C$ ? If so, write congruence statements (such as $\triangle A B C \cong \triangle U V W$ ) for the congruent triangles. (Remember the order of the letters matters!)
4. Are any of the triangles similar but not congruent to $\triangle A B C$ ? If so, write similarity statements (such as $\triangle A B C \cong \triangle U V W$ ) for the similar triangles.
5. According to the table of angle measures, the sum of the angle measures for $\triangle P Q R$ is not 180 degrees. Why do you think this is?

## Properties of Quadrilaterals: Part 2

Name $\qquad$ Date $\qquad$
Complete the following table using a "D" (for definition) for any property that is a part of the definition of the polygon and checking off all the other polygons that have each property. (See the first row for an example.)

|  | Properties |  |  |  |  | $\begin{aligned} & \frac{0}{0} \\ & \frac{0}{0} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \text { 응 } \\ & \text { N } \\ & \text { O } \\ & \text { 인 } \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Polygon with four sides | D | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
|  | Both pairs of opposite sides must be \||. |  |  |  |  |  |  |  |
|  | All angles must be right angles. |  |  |  |  |  |  |  |
|  | All sides must be $\cong$. |  |  |  |  |  |  |  |
|  | Exactly one pair of opposite sides is \||. |  |  |  |  |  |  |  |
|  | Exactly one pair of opposite sides is $\cong$. |  |  |  |  |  |  |  |
|  | Two pair of adjacent sides must be $\cong$. |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | Both pairs of opposite sides must be $\cong$. |  |  |  |  |  |  |  |
|  | Both pairs of opposite angles must be $\cong$. |  |  |  |  |  |  |  |
|  | All angles must be $\cong$. |  |  |  |  |  |  |  |
|  | Diagonals must bisect each other. |  |  |  |  |  |  |  |
|  | Diagonals must be $\cong$. |  |  |  |  |  |  |  |
|  | Diagonals must be $\perp$. |  |  |  |  |  |  |  |
|  | Diagonals bisect opposite angles. |  |  |  |  |  |  |  |
|  | Consecutive angles must be supplementary. |  |  |  |  |  |  |  |



## Properties of Quadrilaterals: Part 3

Name $\qquad$ Date $\qquad$
Use your knowledge of quadrilaterals to solve the following problems.

1. You want to build a plant stand with three equally spaced circular shelves. You want the top shelf to have a diameter of 6 inches and the bottom shelf to have a diameter of 15 inches. The diagram at the right shows a vertical cross-section of the plant stand. What is the length of the middle shelf?

2. Prove that the quadrilateral shown below on the grid is a parallelogram. Show all work.


## Zombie Quadrilateral

Name $\qquad$ Date $\qquad$
Breaking News! It has been reported that the principal of your school has been infected by the zombie virus. The entire town is at risk for a zombie outbreak.

Your principal was last seen chasing joggers in the park. You and your friends need to find the fastest routes to safety. You find a map of your town in a kitchen drawer. Use the map to determine which routes you and your friends should take for each situation.


1. You are at the school and need to get to safety. You have the choice of running down Pine Street or up Ridge Street. Which is a shorter distance? What does this tell you about Elm Street and First Street? Explain your answer.
2. You are finally safe at your house when you hear the breaking news that the zombies are heading toward your friend's house. Your friend wants to run to your house because he is afraid to be home alone. He thinks it is safer to run on the roads, but you say it is a shorter distance to run a straight diagonal from each house. Who is correct? What is the difference in distance? What does this tell you about diagonals in a quadrilateral? Justify your answer.
3. Betty is at the school and wants to join you and your friend in the safety of your house. She is not sure how to get to your house and her phone is dead, so she cannot use GPS. Zombies have been reported walking down Main Street by the park. Describe two different routes Betty can take that do not include her travelling down the part of Main Street above the park. What is the distance of each route?
4. Billy escaped zombie joggers in the park and is at West Street and South Street. He wants to run to the mall where other non-infected townspeople have gathered for safety. He has two options: a.) Run through the woods up Trail 1 and Trail 2, then head east on North Street. b.) Run east on South Street, then head north up East End Street. Which is the shortest route? Justify your reasoning mathematically and with words.
