These are questions involving skills you need to know from previous math classes. Show evidence of understanding for each problem. We will go over any questions you have the first week of school, and our first quiz will be over this material. Use the suggested websites and notes from previous classes if you need help.

Helpful Websites:
www.algebrahelp.com
www.mathtv.com
www.purplemath.com

FACTORING

Factor completely:
1. $x^2 - 5x + 6$
2. $5x^3 - 45x$
3. $3x^2 - 16x - 12$
4. $3x^3 + 15x - x - 5$
5. $x^3 - 27$
6. $8x^3 + 1$

LINEAR EQUATIONS

1. $(3, -1)$ is the midpoint between $(-2, -7)$ and what other point?

2. Find the distance between $(-2, -7)$ and $(3, -1)$.

3. Write the equation of a line in point slope form that goes through $(2, -4)$ and $(-1, 0)$. Then convert into slope intercept form.

4. Convert the standard linear equation $\frac{y}{2} + \frac{x}{6} = 1$ to slope intercept form.
GRAPHING TRANSFORMATIONS OF FUNCTIONS

Quadratic functions and other polynomials can be graphed by determining the \( x \) and \( y \) intercepts of the graph and its expected shape as it relates to its parent function. Your study of algebraic functions has included families of functions such as the absolute value graph, cubic functions, square functions, and others. Transformations of these familiar graphs result in new graphs that are similar to the parent graph. Changes to a parent graph can involve vertical or horizontal shifts as well as reflections over either axis.

- The graph of the function \( y = f(x) \) is shifted vertically by \( y = f(x) + k \). The graph is shifted up if \( k > 0 \) and down if \( k < 0 \).
- The graph of the function \( y = f(x - h) \) is shifted horizontally: left if \( h < 0 \) and right if \( h > 0 \).
- The graph of the function \( y = f(x) \) is reflection about the \( x \)-axis with \( y = -f(x) \).
- The graph of the function \( y = f(x) \) is reflection about the \( y \)-axis with \( y = f(-x) \).

For additional help [http://mathforum.org/library/topics/polynomials/](http://mathforum.org/library/topics/polynomials/)

(1) Consider the graph at the left as \( y = h(x) \).

Show each variation on the given coordinate grid.

(a) \( y = h(x) - 3 \)

(b) \( y = h(x + 2) \)

(2) Consider the graph at the left as \( y = g(x) \).

Show each variation on the given coordinate grid.

(a) \( y = -g(x) \)

(b) \( y = -g(x - 2) \)
The intercepts and shape of a graph are determined by the degree of the polynomial and the specific factors of the algebraic expression. This quartic polynomial has a degree of four and a positive leading coefficient.

One can "read" the domain and range from the graph. The domain is the set of x values that can be considered as input value of the function. The range is the set of y values that are determined by the output of the function.

The domain of the graph above is all real numbers, or \((-\infty, \infty)\). The range is the y-values that are greater than or equal to the symmetric minimum points. If one of those minimum points were \((5, -8)\), then the domain is \([-8, \infty)\).

(3) Identify the domain and range of each graph. Write each in interval notation. Use reasonable values for max and min points.

a) domain ________  b) domain ________  c) domain ________
  range ________  range ________  range ________

d) domain ________  e) domain ________  f) domain ________
  range ________  range ________  range ________

- y-intercept: \((0, 12)\)
GRAPHING FUNCTIONS

Purpose: To practice graphing functions, arrive at a clearer understanding of what functions are by examining their characteristics, and to become more familiar with the usefulness of your graphing calculator.

This assignment consists of the following:

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<th>Finding intersection points</th>
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<td>Locating zeros and understanding their connection with the degree of the polynomial</td>
<td>Exploring end behavior and Identifying local maximum/minimum</td>
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Helpful Websites:

http://www.prenhall.com/divisions/esm/app/graphing/ti83/

Example A: This is a non-calculator problem.
Find the zeros of the polynomial written in factored form:  \( y = (x + 2)(x - 1)(x - 4) \)

Solution: You find the zeros by taking each expression, setting it equal to zero, and solving for \( x \).

\[
egin{align*}
  x + 2 &= 0 & x - 1 &= 0 & x - 4 &= 0 \\
  x &= -2 & x &= 1 & x &= 4
\end{align*}
\]

The zeros of the polynomial are \( x = -2, 1, 4 \)
This means that the graph will touch the \( x \)-axis at \((-2, 0), (1, 0), \) and \((4, 0)\).

Practice: For each of the examples, find the zeros.

1. \( y = (x + 5)^2(2x - 1) \)
2. \( y = (x^2 - 25)(x + 2i)(x - 2i) \)
Example B:
Use the graphing calculator to graph polynomials, set windows, and locate zeros.
Graph: \( y = -(x+12)(x-11) \) Give an example of a proper window.

Solution: The graph of this function is a parabola flipped over the x-axis. The zeros are \( x = -12 \) and \( 11 \), so we need to set a window that will show where this graph touches the x-axis. The y-intercept, found by plugging zero in for \( x \), is 132. Our maximum value of \( y \) must be greater than 132. One proper window would be where \( x: [-15, 15] \), \( y: [-10, 140] \).

Practice: For each of the examples, sketch the polynomial on graph paper and give the window required to view the polynomial.

3. \( y = x^2 - 4x + 20 \)
   Window \( x: [\quad , \quad ] \)
   \( y: [\quad , \quad ] \)

4. \( y = (x + 4)(x + 1)(x - 3) \)
   Window \( x: [\quad , \quad ] \)
   \( y: [\quad , \quad ] \)

5. \( y = -x^2 + x + 12 \)
   Window \( x: [\quad , \quad ] \)
   \( y: [\quad , \quad ] \)

6. \( y = x^4 - 2x^3 - 16x^2 + 2x + 15 \)
   Window \( x: [\quad , \quad ] \)
   \( y: [\quad , \quad ] \)

Example C:
This is a non-calculator problem.

Solve using the quadratic formula. Quadratic Formula:
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\( y = x^2 + 4x - 7 \) \( a = 1 \), \( b = 4 \), and \( c = -7 \)

Step 1. \( x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-7)}}{2(1)} \) Step 2. \( x = \frac{-4 \pm \sqrt{16 + 28}}{2} \) Step 3. \( x = \frac{-4 \pm \sqrt{44}}{2} \)
Step 4. \( x = \frac{-4 \pm \sqrt{4i \cdot 11}}{2} \) Step 5. \( x = \frac{-4 \pm 4i\sqrt{11}}{2} \) Step 6. \( x = -2 \pm 2i\sqrt{11} \)

Practice: Solve using the Quadratic Formula. Reduce radicals, if there is a negative number underneath the radical, use imaginary numbers "i" to simplify.

7. \( y = x^2 - 6x - 3 \)
8. \( y = (x - 3)^2 \)
9. \( y = 2x^2 - 4x + 4 \)
Example D: For each of the polynomials:
   a. Give the degree and zeros
   b. Discuss the end behavior
   c. Find the domain and range
   d. Identify the y-intercept

\[ y = x^3 + x^2 - 9x - 9 \]

Solution:
   a. This is a degree 3 polynomial (cubic)
      The zeros are \( x = -3, -1, 3 \) (as seen on the graph)
   b. \( \lim_{{x \to -\infty}} f(x) = \infty \) and \( \lim_{{x \to \infty}} f(x) = -\infty \)
   c. The domain and range are all real numbers expressed in interval notation:
      \((-\infty, \infty)\)
   d. The y-intercept is (0, -9). If you look at the equation, it is the constant term.

Practice:
10. \( y = x^2 - 4x + 20 \)               11. \( y = (x + 4)(x + 1)(x - 3) \)
12. \( y = -x^2 + x + 12 \)               13. \( y = x^4 - 2x^3 - 16x^2 + 2x + 15 \)

Example E: Graph each pair of functions and then find their intersection point(s).
1. \( y = x^2 - 4x - 5 \) and \( y = \frac{1}{2} x + 1 \)

Solution: Type both functions into your calculator. Graph. You will see a parabola and a line touching twice. We need to find both of these intersection points.
Press 2nd Calc-7 #5 Intersection-7 make sure cursor is on one curve-7 Enter-7 make sure cursor is on the other curve-7 Enter-7 move cursor to where the intersection point is-7 Enter
The intersection point will be displayed on the bottom of the screen (-1.076, 0.462)
Following the same steps, you should find the second intersection point to be (5.576, 3.788)

Practice:
14. \( y = 2x - 1 \) and \( y = x^2 - 2x - 3 \)
Practice:
15. A model for the height of a toy rocket shot from a platform is \( y = -16x^2 + 145x + 7 \), where \( x \) is the time in seconds and \( y \) is the height in feet.
   a. Graph the function
   b. Find the zeros of the function.
   c. What do the zeros represent? Are they realistic?
   d. About how high does the rocket fly before hitting the ground? Explain.

16. The volume in cubic feet of a box can be expressed as \( V(x) = x^3 - 6x^2 + 8x \), or as the product of three linear factors with integer coefficients. The width of the box is \( 2 - x \).
   a. Factor the polynomial to find linear expressions for the height and length.
   b. Graph the function. Find the \( x \) – intercepts. What do they represent?
   c. Describe a realistic domain for the function.
   d. Find the maximum volume of the box.

TRUE / FALSE. If the statement is false, explain why.

17. T / F No polynomial equation can have an odd number of imaginary roots.

18. T / F A 4th degree polynomial will have no more than 4 real zeros.

19. T / F A linear term has degree one.

20. T / F To find the y-intercept, one would plug zero in for \( x \).
SPECIAL RIGHT TRIANGLES

There are two triangles whose side lengths follow specific relationships. One is created by a reflection line in an equilateral triangle; the other is an isosceles right triangle. The lengths of the legs of a 30°-60°-90° triangle are created by multiplying or dividing by $\sqrt{3}$. The factor that relates the short leg and the hypotenuse is 2. The lengths of the legs of a 45°-45°-90° triangle are equal while the hypotenuse is larger by a factor of $\sqrt{2}$. Notice the relationships shown in each diagram below.

Additional explanations and examples can be found at http://www.cliffsnotes.com/WileyCDA/CliffsReviewTopic/Special-Right-Triangles.

Find the missing sides of each triangle shown:
TRIG RATIOS

Specific ratios are defined to explain the relationship between the legs and the hypotenuse of a right triangle. Consider the terms that are used to identify the triangle segments with respect to the angle $\theta$.

\[
\tan \theta = \frac{\text{opposite leg}}{\text{adjacent leg}}
\]

\[
\sin \theta = \frac{\text{opposite leg}}{\text{hypotenuse}}
\]

\[
\cos \theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}
\]

The appropriate trig ratio creates a ratio for the angle in the triangle.

\[
\sin \theta = \frac{5}{13}
\]
\[
\sin \theta \approx 0.385
\]

As you consider the triangle at the left, choose the correct trig ratio for the triangle sides that are marked. Using your calculator, you can find an approximation for $x$.

\[
\sin 78^\circ = \frac{x}{18} \quad \left( \frac{\text{opposite}}{\text{hypotenuse}} \right)
\]

\[
18 \sin 78^\circ = x
\]
\[
x \approx 17.61
\]
13 is adjacent to the 62° angle. Write an equation, then use your calculator to find x.

\[
\cos 62° = \frac{13}{x} \\
x \cos 62° = 13 \\
x = \frac{13}{\cos 62°} \approx 27.69
\]

A trig ratio and the inverse key of your calculator give us a way to find the missing angle of a triangle.

\[
\cos \theta = \frac{8}{12} \\
\cos \theta \approx 0.667 \\
\theta = \cos^{-1} \frac{8}{12} \\
\theta \approx 48.19°
\]

Write and solve trig equations to find the missing sides or angles of each.