

NAME: _____

PRECALCULUS TEACHER: _____

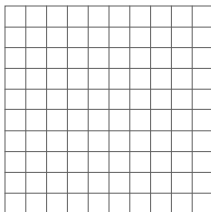
**LAKE BRADDOCK
HIGH SCHOOL
CALCULUS SUMMER
REVIEW PACKET
2009**

**FOR STUDENTS ENTERING
AP CALCULUS AB/BC IN
SEPT. 2009**

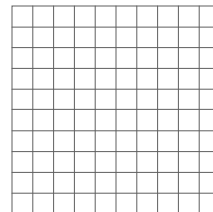
WS 1.1, Four Ways to Represent a Function

Ex 1. Sketch the graph and find the domain and range of each function.

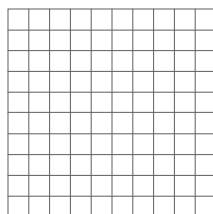
a) $f(x) = 2x - 1$



b) $g(x) = x^2$

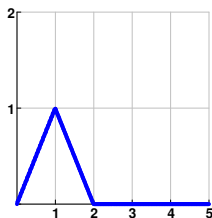


Ex 2. When you turn on a hot water faucet, the temperature T of the water depends on how long the water has been running. Draw a rough graph of T as a function of the time t that has elapsed since the faucet was turned on.



Ex 3. A rectangular storage container with an open top has a volume of 10 m^3 . The length of its base is twice its width. Material for the base cost \$10 per square meter; material for the sides cost \$6 per square meter. Express the cost of materials as a function of the width of the base.

Ex 4. Find a formula for the function f graphed below.



Ex 5. Determine whether each of the following functions is even, odd, or neither. Explain.

a) $f(x) = x^5 + x$



b) $g(x) = 1 - x^4$



c) $h(x) = 2x - x^2$

Ex 6. (1973AB 7) Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y = \frac{x+1}{x}$

(B) $y = -x^5 + 3x$

(C) $y = x^4 - 2x^2 + 6$

(D) $y = (x-1)^3 + 1$

(E) $y = (x^2 + 1)^2 - 1$

Ex 7. (1973AB 12) If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) -6

(B) -3

(C) -1

(D) 2

(E) It cannot be determined from the information given

Ex 8. (1985AB 26) The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

I. The x -axis

II. The y -axis

III. The origin

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

Ex 9. (1988AB 2) What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x: x \neq 3\}$

(B) $\{x: |x| \leq 2\}$

(C) $\{x: |x| \geq 2\}$

(D) $\{x: |x| \geq 2 \text{ and } x \neq 3\}$

(E) $\{x: x \geq 2 \text{ and } x \neq 3\}$

Ex 10. (1988AB 44) Let f and g be odd functions. If p , r , and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$

II. $r(x) = f(x) + g(x)$

III. $s(x) = f(x)g(x)$

(A) I only

(B) II only

(C) I and II only

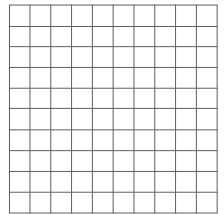
(D) II and III only

(E) I, II, and III

WS 1.2, Mathematical Models

Ex 1. a) As dry air moves upward, it expands and cools. If the ground temperature is 20°C and the temperature at a height of 1 km is 10°C , express the temperature T (in $^{\circ}\text{C}$) as a function of height h (in km), assuming that a linear model is appropriate.

b) Draw the graph of the function in part (a). What does the slope represent?



c) What is the temperature at a height of 2.5 km?

Year	CO ₂ level (in ppm)
1972	327.3
1974	330.0
1976	332.0
1978	335.3
1980	338.5
1982	341.0
1984	344.3
1986	347.0
1988	351.3
1990	354.0

Ex 2. The table lists the average carbon dioxide level in the atmosphere, measured in parts per million at Mauna Loa Observatory from 1972 to 1990. Use the data to find a model for the carbon dioxide level.

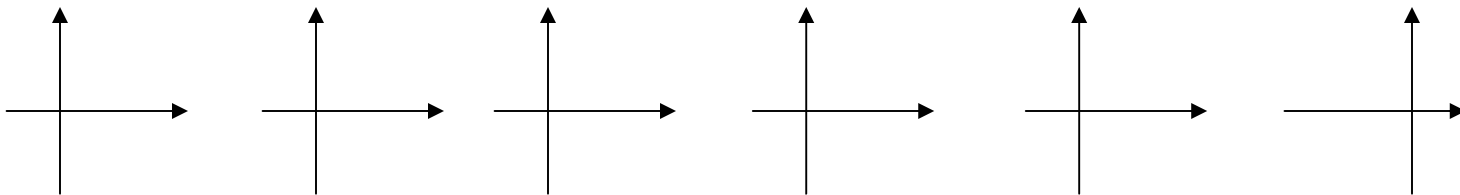
Ex 3. Use the model in Ex. 2 to estimate the average CO₂ level for 1987 and to predict the level for the year 2005. According to this model, when will the CO₂ level exceed 400 ppm?

Time (sec)	Height (meters)
0	450
1	445
2	431
3	408
4	375
5	332
6	279
7	216
8	143
9	61

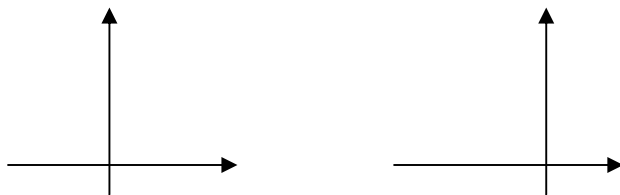
Ex 4. A ball is dropped from the upper observation deck of the CN Tower, 450m above the ground, and its height h above the ground is recorded at 1-second intervals on the table to the left. Find a model to fit the data and use the model to predict the time at which the ball hits the ground.

WS 1.3, New Functions from Old Functions

Ex 1. Given the graph of $y = \sqrt{x}$, use transformations to graph $y = \sqrt{x-2}$, $y = \sqrt{x-2}$, $y = -\sqrt{x}$, $y = 2\sqrt{x}$, and $y = \sqrt{-x}$.



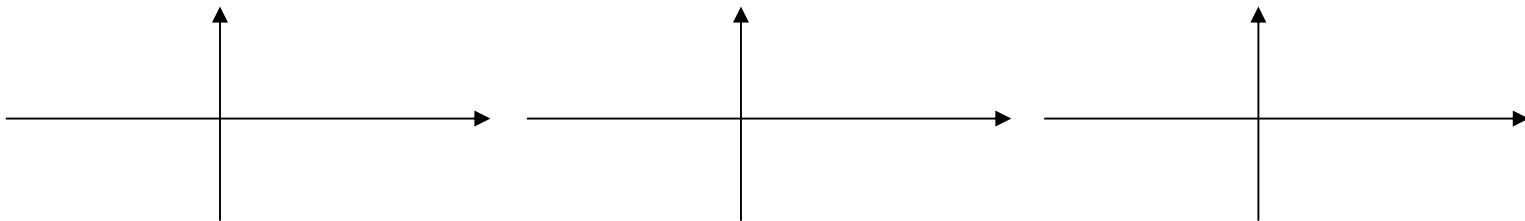
Ex 2. Sketch the graph of the function $f(x) = x^2 + 6x + 10$.



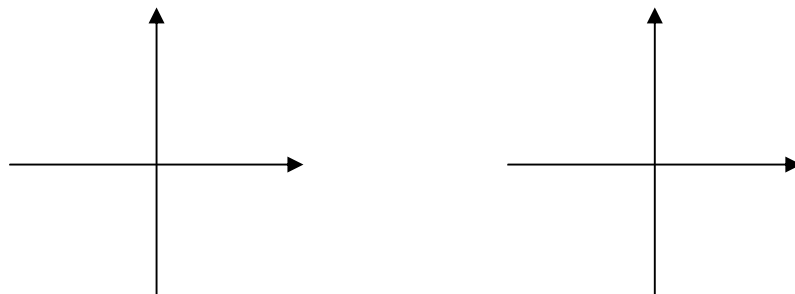
Ex 3. Sketch the graphs of the following functions.

a) $y = \sin 2x$

b) $y = 1 - \sin x$



Ex 4. Sketch the graph of the function $y = |x^2 - 1|$.



Ex 5. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4-x^2}$, find the functions $f + g$, $f - g$, fg , and f/g .

Ex 6. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find each function and its domain.

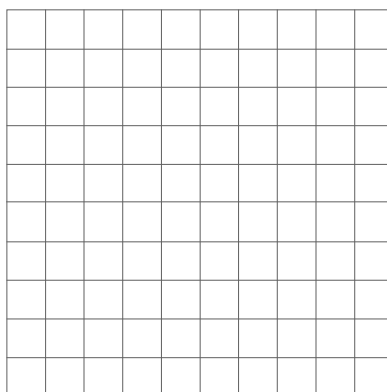
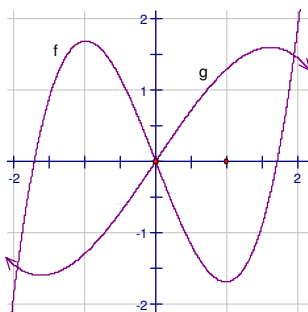
a) $f \circ g$

b) $g \circ f$

c) $f \circ f$

d) $g \circ g$

Ex 7. The graphs of f and g are shown below and $h = f \circ g$. Estimate the value of $h(0.5)$. Then sketch the graph of h .



Ex 8. (1993AB 9) If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

(A) $3x^3 - |x|$

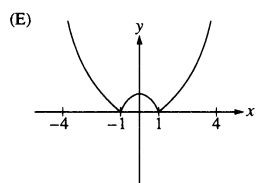
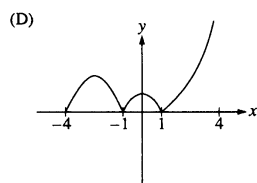
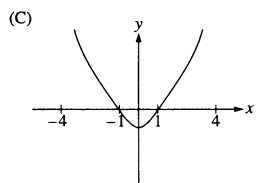
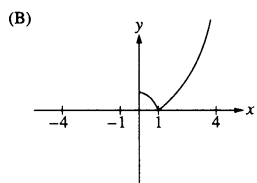
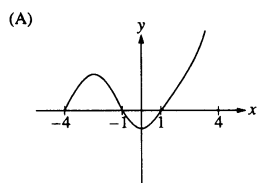
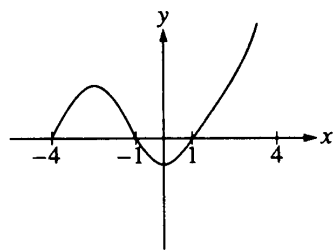
(B) $|3x^2 - 1|$

(C) $3x^2|x| - 1$

(D) $3|x| - 1$

(E) $3x^2 - 1$

Ex 9. (1993AB 40) The graph of $y = f(x)$ is shown in the figure below. Which of the following could be the graph of $y = f(|x|)$?



WS 1.5, Exponential Functions

Laws of exponents If a and b are positive numbers and x and y are any real numbers, then

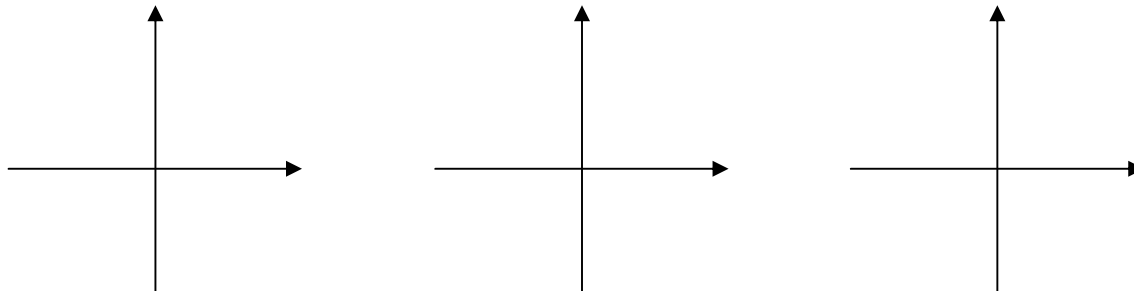
$$1. a^{x+y} = a^x a^y$$

$$2. a^{x-y} = \frac{a^x}{a^y}$$

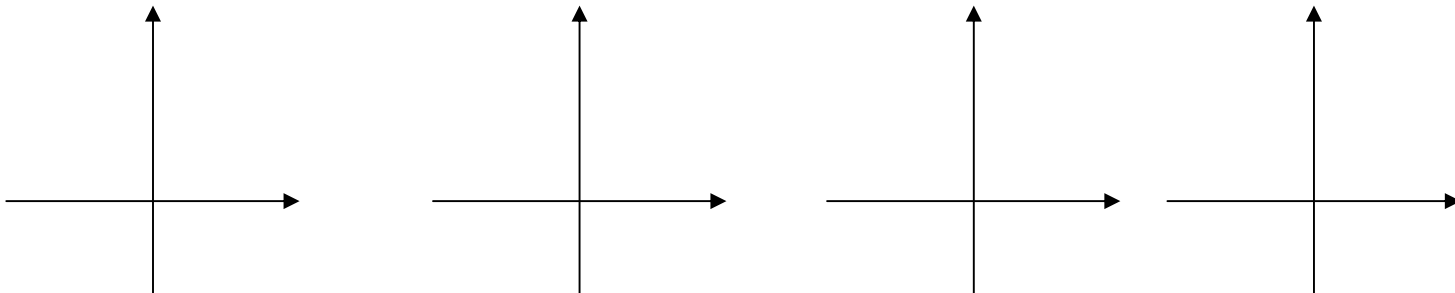
$$3. (a^x)^y = a^{xy}$$

$$4. (ab)^x = a^x b^x$$

Ex 1. Sketch the graph of the function $y = 3 - 2^x$ and determine its domain and range.



Ex 2. Graph the function $y = \frac{1}{2}e^{-x} - 1$ and state the domain and range.



Ex 3. Use a calculator to find the values of x for which $e^x > 1,000,000$. (Indicate the window dimensions that you used).

Ex 4. (1973AB 5) If $f(x) = e^x$, which of the following lines is an asymptote to the graph of f ?

- (A) $y = 0$ (B) $x = 0$ (C) $y = x$ (D) $y = -x$ (E) $y = 1$

WS 1.6, Inverse Functions and Logarithms

Definition A function f is called a **one-to-one function** if it never takes on the same value twice; that is

$$f(x_1) \neq f(x_2) \text{ whenever } x_1 \neq x_2$$

Horizontal Line Test A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Ex. 1. Is the function $f(x) = x^3$ one to one? Explain.

Ex. 2. Is the function $g(x) = x^2$ one to one? Explain.

Definition Let f be a one-to-one function with domain A and range B . Then its **inverse function** f^{-1} has domain B and range A and is defined by

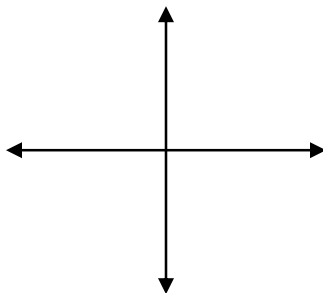
$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

for any y in B .

Ex. 3. If $f(1) = 5$, $f(3) = 7$, and $f(8) = -10$, find $f^{-1}(7)$, $f^{-1}(5)$, and $f^{-1}(-10)$.

Ex. 4. Find the inverse function of $f(x) = x^3 + 2$.

Ex. 5. Sketch the graphs of $f(x) = \sqrt{-1-x}$ and its inverse function using the same coordinate axes.



Ex. 6. Use the laws of logarithms to evaluate $\log_2 80 - \log_2 5$.

Ex. 7. Find x if $\ln x = 5$.

Ex. 8. Solve the equation $e^{5-3x} = 10$.

Ex. 9. Express $\ln a + \frac{1}{2} \ln b$ as a single logarithm.

Ex 10. (1973AB 31) If $\log_a(2^a) = \frac{a}{4}$, then $a =$
(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

Ex 11. (1988AB 22) If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$
(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

Ex 12. (1988AB 31) If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1} , is given by $f^{-1}(x) =$
(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) x