Chantilly High School
AP Calculus BC

To: All students enrolled in AP Calculus AB for the school year 2009 – 2010
From: AP Calculus BC teacher: Ms. St. Clair

Going into AP calculus, there are certain skills that have been taught to you over the previous years that I assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

I assume that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example, should now be a part of you. If not, you would not be going on to AP calculus. The topics covered in the packet are skills that are used continually in AP calculus.

The attached summer packet is for all students enrolled in AP Calculus BC in the fall. This packet will be collected on the first day of school and graded. Show all work on separate paper. NO work written in the question packet will be checked.

Rather than give you a textbook to remind you of the techniques necessary to solve the problem, I have given you several websites that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, examine these websites. Don't fake your way through these problems. As stated, students are notoriously weak in them, even students who have achieved well prior to AP calculus. Use the websites.

Realize also that certain concepts are interrelated. Domain, for example, may require you to be an expert at working with inequalities. Solving quadratic equations may involve techniques used in solving fractional equations.

I have included the prerequisites for AP Calculus BC on the next page. Look over the list and be sure you are comfortable with all of the topics.

This packet is due the first day back in school in the fall. It will be graded. You will have a test on this material the second week of school. You need to get off to a good start so spend some quality time on this packet this summer. Do not rely on the calculator. Use the calculator only on the problems where calculator use is indicated. Half of the AP exam next year is taken without a calculator.

It is a mistake to decide to do this now. Let it go until mid-summer. I want these techniques to be relatively fresh in your mind in the fall. Also, do not wait to do them at the very last minute. These take time.

If you have any questions about any of these problems or techniques used in solving them, you may contact me at the school website/email address. Have a good summer and see you in the fall.

Have a great summer!

Prerequisite Skills for Calculus BC

Before studying calculus, all students should complete a full four-year preparation of secondary mathematics designed for college-bound students: courses in which they study Algebra, Geometry, Trigonometry, and Precalculus (analytic geometry and elementary functions).

Students must be familiar with functions that include:

Linear	Polynomial	Rational
Exponential	Logarithmic	Trigonometric
Inverse Trigonometric	Piecewise defined	

Students must be familiar with:

- The properties of functions
- The algebra of functions
- The graphs of functions

Students must also understand the language of functions:

Domain and Range	odd and even	periodic
Symmetry	zeros	intercepts

Know the exact values of the trigonometric functions of common angles such as $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

Students must be familiar with precalculus topics.

1. Functions, Graphs and Limits
   Analysis of graphs – using technology and calculus to predict and to explain the observed local and global behavior of a function.

2. Limits of functions (including one-sided limits)
   Calculating limits from graphs or tables of data or using algebra.

3. Asymptotic and unbounded behavior.
   Understanding asymptotes in terms of graphical behavior, describing asymptotic behavior in terms of limits involving infinity, comparing relative magnitudes of behavior of functions and their rates of change, and understanding the very significant behavior of functions that the asymptotes reveal.

4. Continuity as a property of functions.
   Understanding continuity in terms of limits and geometric understanding of graphs of continuous functions.

5. Parametric, polar, and vector functions.

6. Derivatives.
   The concept of the derivative presented geometrically, numerically, and analytically, and is interpreted as an instantaneous rate of change. Understand the definition of a derivative as the limit of the difference quotient. Understand the relationship between differentiability and continuity.

7. Derivative at a point.
   Understand the slope of a curve at a point, the tangent line to a curve at a point, and local linear approximation. Understand instantaneous rate of change as the limit of average rate of change. Approximate rate of change from graphs and tables of values.

8. Derivative as a function.
   Corresponding characteristics of graphs of $f(x)$ and $f'(x)$. Relationship between the increasing and decreasing behavior of $f(x)$ and the sign of $f'(x)$.

Below are listed topics in the review. You can certainly do Google searches for any of these topics. But I have given you several sites that will cover pretty much all of these topics.

Here is a good site for most algebra topics:

http://www.purplemath.com/modules/index.htm

Beginning algebra topics

Exponents

Negative and fractional exponents

Intermediate algebra topics

Domain
Solving inequalities: absolute value
Solving inequalities: quadratic
Special Factoring formulas
Function transformation
Factor theorem (*p* over *q* method)
Even and odd functions
Solving quadratic equations and quadratic formula

Advanced algebra topics

Asymptotes
Complex fractions
Composition of functions
Solving Rational (fractional) equations

Trig Information

http://www.mathematicshelpcentral.com/index.html

Once in the site, go to lecture notes.

Basic right angle trig
Trig equations

Limits

http://www.calculus-help.com/funstuff/phobe.html

CHANTILLY HIGH SCHOOL
AP CALCULUS BC SUMMER ASSIGNMENT Name_____

Complete the following. <u>Show and attach all work in a clear manner</u>. Do NOT do any work on this sheet – all work must be separate. Have this assignment completed and ready to turn in on the first day of school.

1 – 6. Are the following statements true? If not, explain in words why not?

1. $\frac{2k}{2x+h}=\frac{k}{x+h}$ 2. $\frac{1}{p+q}=\frac{1}{p}+\frac{1}{q}$ 3. $\frac{x+y}{2}=\frac{x}{2}+\frac{y}{2}$

4. $3\frac{a}{b}=\frac{3a}{3b}$ 5. $3\frac{a}{b}=\frac{3a}{b}$ 6. $3\frac{a+b}{c}=\frac{3a+b}{c}$

7 – 16. Factor each of the following completely.

7. a^2-b^2 8. a^3-b^3 9. $8x^3+y^3$ 10. $4x^2-21x-18$

11. $2x^2+x-3$ 12. $3x^2+6x^3-9x$ 13. $(x+1)^3(4x-9)-(16x+9)(x+1)^2$

14. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$ 15. $(2x-1)^2(x-3)+(x+1)(2x-1)^3$

16. Factor $x-a$ in such a way that $\sqrt{x}-\sqrt{a}$ is a factor.

17 - 24. Simplify:

17. $\frac{x/2}{x/4}$ 18. $h\div\frac{(x+h)}{h}$ 19. $\frac{\sqrt{x-2}+\frac{5}{\sqrt{x-2}}}{x-2}$ 20. $\frac{x(5x+1)-3(x^2+1)}{(x-1)^2}$

21. $\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)^3}$ 22. $\frac{3x(x+1)-2(2x+1)}{(x-1)^2}$

23. $\frac{2x(x+1)^2-3(x+1)^3}{8x^3+30x^2+18x}$ 24. $\frac{(x-1)^3(2x-3)-(4x-1)(x-1)^2}{(x-1)^2(2x-1)}$

25 – 28. Solve the equation.

25. $4x^2-21x-18=0$ 26. $x^3+3x^2-5x-15=0$ 27. $x^4-9x^2+8=0$ 28. $4-3^x=0$

29. Write as a single fraction with denominator in factored form: $\frac{7x^2+5x}{x^2+1}-\frac{5x}{x^2-6}=0$

30. Graph the equation $y=x^3-x$ and answer the following questions.
   a) Is the point (3, 2) on the graph?
   b) Is the point (2, 6) on the graph
   c) Is the function even, odd, or neither?
   d) What is the y intercept?
   e) Find the x intercepts.

31. Find all intercepts of the graph of $y = \dfrac{x-1}{x+3}$

32 – 35. Show work to determine if the relation is even, odd, or neither.

32. $f(x) = 2x^2 - 7$ 33. $f(x) = -4x^3 - 2x$ 34. $f(x) = 4x^2 - 4x + 4$ 35. $f(x) = \dfrac{x^2}{x^2 - 4}$

36. Find the equation of the straight line that passes through the point (2, 4) and is parallel to the line 2x + 3y – 8 = 0.

37. Find the equation of the line that is perpendicular to the line 2x + 3y – 8 = 0 at the point (1,2)

38. The line with the slope 5 that passes through the point (-1,3) intersects the x axis at a point. What are the coordinates of this point?

39. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2,4) intersects the y axis?

40. Given $f(x) = |x-3| - 5$ find $f(1) - f(5)$.

41. Find all points of intersection of the graphs of $x^2 + 3x - y = 3$ and $x + y = 2$

42. If the point $(-1, 1)$ lies on the graph of the equation $kx^2 - xy + y^2 = 5$, find the value of k.

43. Write the equation of a graph that is a function.

44. Write the equation of a graph that is not a function.

45 – 48. Find the domain for each of the following functions.

45. $h(x) = \dfrac{1}{4x^2 - 21x - 18}$ 46. $k(x) = \sqrt{x^2 - 5x - 14}$

47. $p(x) = \dfrac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$ 48. $y = \ln(2x - 12)$

49. For the function $y = 5 - \sqrt{9 - x^2}$, a) find the domain, b) find the range, and c) determine whether the function is odd, even, or neither.

50. Let $f(x) = \begin{cases} -0.5x & x < -2 \\ \sqrt{x+2} & x \geq -2 \end{cases}$ a) draw the graph of *f(x)*. b) find the domain, c) find the range.

51. Find $f(x + \Delta x)$ for $f(x) = x^2 - 2x - 3$.

52 – 53. Sketch the graph of each function

52. $f(x) = \begin{cases} 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$

53. $f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x + 3 & (2, \infty) \end{cases}$

54. State the domain, range and intercepts of the function $y = 2^{-x} - 1$.

55 – 57. Given $f(x) = x - 3$ and $g(x) = \sqrt{x}$ complete the following

55. $f(g(x)) =$ 56. $g(f(x)) =$ 57. $f(f(x)) =$

58 – 60. Given $f(x) = \dfrac{1}{x-5}$ and $g(x) = x^2 - 5$ complete the following

58. $f(g(7)) =$ 59. $g(f(v)) =$ 60. $g(g(x)) =$

61 – 64. Let $f(x) = 2x - 2$. Complete the following:

61. Sketch the graph of $f(x)$.
62. Determine whether *f* has an inverse function.
63. Sketch the graph of $f^{-1}(x)$
64. Give the equation for $f^{-1}(x)$

65 – 66. Simplify using only positive exponents. Do not rationalize the denominator.

65. $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ 66. $\left(\dfrac{1}{x^{-2}} + \dfrac{4}{x^{-1}y^{-1}} + \dfrac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

67 – 72. If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of *f(x)*.

67. $f(x) - 4$ 68. $f(x-4)$ 69. $-f(x+2)$

70. $5f(x) + 3$ 71. $f(2x)$ 72. $|f(x)|$

73. The dollar value of a product in 1998 is $78. The value of the product is expected to decrease $5.75 per year for the next 5 years. Write a linear equation that gives the dollar value V of the product in terms of the year t. (Let t = 8 represent 1998).

74. A business had annual retail sales of $124,000 in 1993 and $211,000 in 1996. Assuming that the annual increase in sales follows a linear pattern, predict the retail for 2007.

75. In order for a company to realize a profit in the manufacture and sale of a certain item, the revenue, R, for selling x items must be greater than the cost, C, of producing x items. If R = 69.99x and C = 59x + 850, for what values of x will this product return a profit?

76. Suppose that in any given year, the population of a certain endangered species is reduced by 25%. If the population is now 7500, in how many years will the population be 4000?

77. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x.

78. a) Graph the parametrized curve described by $x = 2\sin t,\ y = -3\cos t,\ 0 \le t \le \pi$. Indicate the direction in which the curve is traced. b) Find a Cartesian equation for the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

79. Let $f(x)=\sqrt[3]{x+2}$ and $g(x)=x^3-2$. Which of the following are true?

I. $g(x)=f^{-1}(x)$ for all real values of x.

II. $(f\circ g)(x)=1$ for all real values of x.

III. The function f is one to one

80. Let $f(x)=\sqrt{3-x}$. Find an expression for $f^{-1}(x)$. (Be sure to state any necessary domain restrictions.)

81. The table gives Taiwan's nuclear power generation data in billions of kilowatt-hours. Let x = 5 represent 1980, x = 10 represent 1985, and so on.

Year	1980	1985	1990	1995
Energy produced	7.8	27.8	31.6	33.9

a. Find a natural logarithm regression equation for the data.

b. predict when Taiwan's nuclear power generation will reach 40 billion kilowatt-hours.

82. An angle measuring $\frac{3\pi}{8}$ radians has its vertex at the center of a circle whose radius is 4 feet. Find the length of the subtended arc.

83. Let $y=3sin(2x-\pi)+2$. Determine the period, domain, and range of the function.

84 – 91. Evaluate: Answers for 90 & 91 must be in radians.

84. $\cos 0$ 85. $\sin 0$ 86. $\tan\frac{\pi}{2}$ 87. $\cos\frac{\pi}{4}$ 88. $\sin\frac{\pi}{2}$

89. $\sin\pi$ 90. $\arccos\frac{\sqrt{3}}{2}$ 91. $\arctan 1$

92 – 94. Find the solution of the equations for $0\le x<2\pi$

92. $2\sin^2\theta=1-\sin\theta$ 93. $2\tan\theta-\sec^2\theta=0$ 94. $\sin 2\theta+\sin\theta=0$

95. Which of the following expressions are identical?

a) $\cos^2 x$ b) $(\cos x)^2$ c) $\cos x^2$

96. Which of the following expressions are identical?

a) $(\sin x)^{-1}$ b) arcsinx c) $\sin x^{-1}$ d) $\frac{1}{\sin x}$

97 – 103. Solve for x.

97. $\ln e^3 = x$

98. $\ln e^x = 4$

99. $\ln x + \ln x = 0$

100. $e^{\ln 5} = x$

101. $\ln 1 - \ln e = x$

102. $\ln 6 + \ln x - \ln 2 = 3$

103. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

104 – 121: Evaluate the limit.

104. $\lim_{x\to 2}(3x^2+5)$

105. $\lim_{x\to -1}\dfrac{x^4+x^3}{x+1}$

106. $\lim_{x\to 25}\dfrac{\sqrt{x}-5}{x-25}$

107. $\lim_{x\to -2}\dfrac{x-4}{x^2-2x-8}$

108. $\lim_{x\to -2}\dfrac{x^3+8}{x+2}$

109. $\lim_{x\to 5}\dfrac{x-5}{|x-5|}$

110. $\lim_{x\to 1} f(x)$ if $f(x)=\begin{cases}3-x & x\neq 1\\ 1 & x=1\end{cases}$

111. $\lim_{x\to -1}\dfrac{x^2+3x+2}{x^2+1}$

112. $\lim_{x\to 2}\sqrt{x^2-4}$

113. If $\lim_{x\to c} f(x) = -\dfrac{1}{2}$, and $\lim_{x\to c} g(x) = \dfrac{2}{3}$, find $\lim_{x\to c}\dfrac{f(x)}{g(x)}$

114. $\lim_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$

115. $\lim_{x\to 2}\sec\dfrac{\pi x}{3}$

116. $\lim_{x\to 0}\dfrac{x}{\tan x}$

117. $\lim_{x\to 3^+}\sqrt{2x-5}$

118. $\lim_{x\to 2^-}\dfrac{1}{x-2}$

119. $\lim_{x\to 2}\dfrac{1}{(x-2)^2}$

120. $\lim_{x\to 0}\left(2+\dfrac{5}{x^2}\right)$

121. $\lim_{x\to\infty}\dfrac{x^2-x-6}{5x^2}$

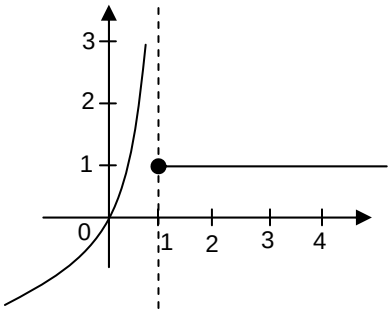
122 – 127. For each of the following determine: a) $\lim_{x\to 1^-} f(x)$ b) $\lim_{x\to 1^+} f(x)$ and c) $\lim_{x\to 1} f(x)$

122. $f(x)=\begin{cases}x^2-1 & x<1\\ 4-x & x\geq 1\end{cases}$

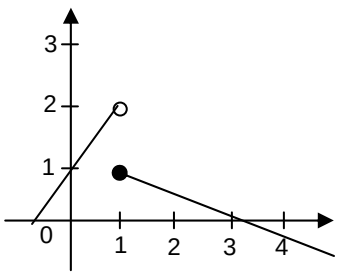
123. $f(x)=\begin{cases}3x-1 & x\leq 1\\ 3-x & x>1\end{cases}$

124. $f(x)=\begin{cases}-x^2 & x<1\\ 2 & x=1\\ x-2 & x>1\end{cases}$

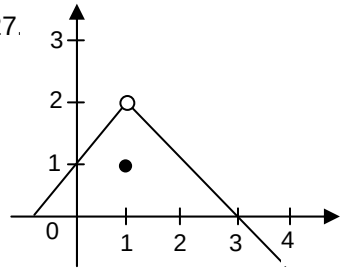
125.



126.



127.



128 – 13. Answer the following.

128. At which values of x is $f(x)=\dfrac{(x+1)(x-3)}{x-2}$ discontinuous? (Be sure to use the definition of continuity).

129. Let $f(x)=\dfrac{1}{x+1}$ and $g(x)=x^2-5$. Find all values of x for which $f(g(x))$ is discontinuous.

130. Determine the value of "c" so that $f(x)$ is continuous on the entire real line. $f(x)=\begin{cases} x-2 & x\le 5 \\ cx-3 & x>5 \end{cases}$

131. Find all vertical asymptotes of $f(x)=\dfrac{x-3}{x+2}$.

132. Find all vertical asymptotes of $g(x)=\dfrac{x+1}{x^2-1}$.

133. $f(x)$ decreases without bound as x approaches what value from the right? $f(x)=\dfrac{4}{(x-3)(5-x)}$

134. Find the derivative of $y=5x^3-7x^2+2x-9$.

135. Find the derivative of $y=\dfrac{x+7}{x^2+1}$.

136. Find the derivative of $y=\sin\left(3x^2\right)$.

137. Find the derivative of $y=\sqrt[4]{2x^3-5x}$.

138. Find the derivative of $y=e^x\cos x$.

Answers

1. No, can not cancel terms. 2. No, can not split the denominator 3. Yes

4. No, 3 is multiplied by fraction not top and bottom 5. Yes 6. No, must distribute 3 to every term

7. $(a-b)(a+b)$ 8. $(a-b)(a^2+ab+b^2)$ 9. $(2x+y)(4x^2-2xy+y^2)$ 10. $(4x+3)(x-6)$

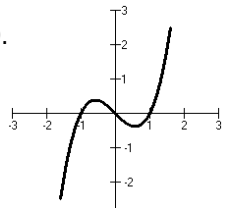
11. $(x-1)(2x-3)$ 12. $3x(x-1)(2x-3)$ 13. $(x+1)^2(4x+3)(x-6)$

14. $(x-1)^2(x+1)(2x-9)$ 15. $2(2x-1)^2(x-1)(x+2)$ 16. $(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})$

17. 2 18. $\frac{h^2}{x+h}$ 19. $\frac{x+3}{(x-2)^{\frac{3}{2}}}$ 20. $\frac{2x-3}{x-1}$ 21. $\frac{4x+3}{x+1}$

22. $\frac{3x+2}{x-1}$ 23. $\frac{-(x+1)^2}{2x(4x+3)}$ 24. $x-4$ 25. x = 6, $x=-\frac{3}{4}$

26. $x=-3,\ x=\pm\sqrt{5}$ 27. $x=\pm1,\ x=\pm2\sqrt{2}$ 28. $x=\frac{\ln 4}{\ln 3}$ 29. $\frac{7x(x^3-6x-5)}{(x^2+1)(x^2-6)}$

30. 

30 a. No 30 b. Yes 30 c. Odd 30 d. (0, 0)

30 e. (1, 0) (-1, 0) (0, 0) 31. $\left(0,-\frac{1}{3}\right)(1,0)$ 32. even 33. odd 34. neither

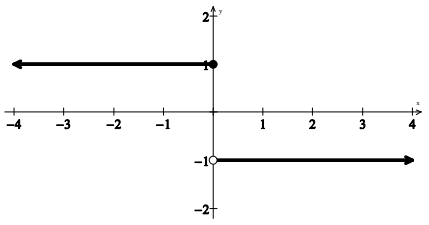
35. even 36. $y-4=-\frac{2}{3}(x-2)$ 37. $y-2=\frac{3}{2}(x-1)$ 38. $\left(-\frac{8}{5},0\right)$ 39. $\left(0,-\frac{2}{3}\right)$ 40. 0

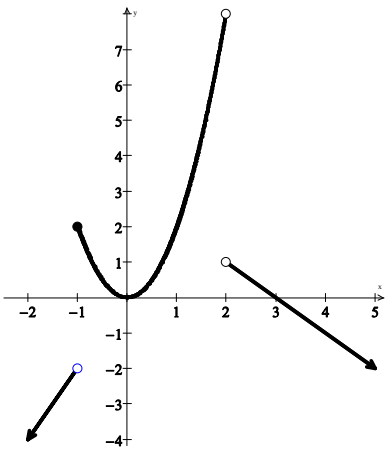
41. $(-5,7)\,(1,1)$ 42. k = 3 43. y = 3 44. x = 3 45. $\left(-\infty,-\frac{3}{4}\right)\cup\left(-\frac{3}{4},6\right)\cup(6,\infty)$

46. $(-\infty,-2]\cup[7,\infty)$ 47. $(-\infty,-5)\cup(6,\infty)$ 48. $(6,\infty)$ 49 a. $[-3,3]$

49 b. $[2,5]$ 49 c. even 50 a. $(-\infty,\infty)$ 50 b. $[0,\infty)$

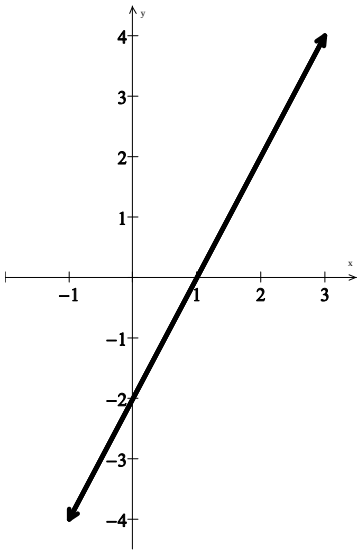
51. $x^2+2x(\Delta x)+(\Delta x)^2-2x-2(\Delta x)-3$

52. 

53. 

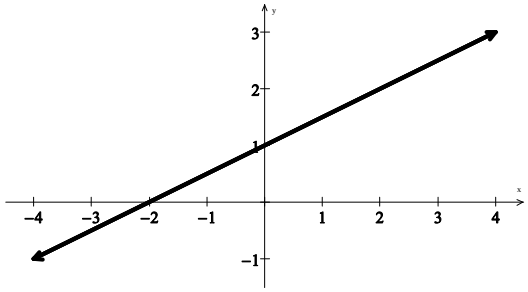
54. Domain: $(-\infty, \infty)$, Range: $(-1, \infty)$, Intercepts: $(0, 0)$ 55. $\sqrt{x} - 3$ 56. $\sqrt{x-3}$

57. x – 6 58. $\frac{1}{39}$ 59. $\frac{-5v^2 + 50v - 124}{(v-5)^2}$ 60. $x^4 - 10x^2 + 20$

61. 

62. Yes 21

64. $y = \frac{1}{2}x + 1$

63. 

65. $\frac{2}{(x-4)^{\frac{1}{4}}}$ 66. $\frac{1}{\sqrt{x^2 + 4xy + y^2}}$ 67. down 4 units 68. right 4 units

69. left 2 units, reflect over x-axis 70. up 3 units, vertical stretch by 5 71. horizontal compression by ½

72. reflects all negative y values over the x-axis 73. $V = -5.75t + 124$ 74. $ 530,000

75. more than 78 items 76. $t = 2.185$ 77. $\frac{x^2}{16} + \frac{(5-x)^2}{4\pi}$

78 a. Right side of ellipse, from (0, -3) to (0, 3) 78 b. $\frac{x^2}{4} + \frac{y^2}{9} = 1; \quad x \geq 0$ 79. I & III

80. $y = -x^2 + 3; \quad x \geq 0$ 81 a. $y = -20.5237 + 19.051 \ln x$ 81 b. 1999 82. $\frac{3\pi}{2}$

83. period: π, range: $[-1, 5]$, Domain: $(-\infty, \infty)$ 84. 1 85. 0 86. undefined

87. $\frac{1}{\sqrt{2}}$ 88. 1 89. 0 90. $\frac{\pi}{6}$ 91. $\frac{\pi}{4}$ 92. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

93. $\frac{\pi}{4}, \frac{5\pi}{4}$ 94. $0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$ 95. a & b 96. a & d 97. 3 98. 4 99. 1

100. 5 101. -1 102. $\frac{1}{3}e^3$ 103. no solution 104. 17 105. -1

106. $\frac{1}{10}$ 107. DNE 108. 12 109. DNE 110. 2 111. 0

112. $\sqrt{5}$ 113. -3 114. $\frac{1}{4}$ 115. -2 116. 1

117. 1 118. $-\infty$ 119. ∞ 120. ∞ 121. $\frac{1}{5}$

122 a. 0 122 b. 3 122 c. DNE 123 a. 2 123 b. 2

123 c. 2 124 a. -1 124 b. -1 124 c. -1 125 a. ∞

125 b. 1 125 c. DNE 126 a. 2 126 b. 1 126 c. DNE

127 a. 2 127 b. 2 127 c. 2 128. $\lim_{x\to 2}\frac{(x+1)(x-3)}{x-2} = DNE$, x = 2

129. $x = \pm 2$ 130. $c = \frac{6}{5}$ 131. $x = -2$ 132. $x = 1$ 133. x = 5

134. $y' = 15x^2 - 14x + 2$ 135. $y' = \frac{-x^2 - 14x + 1}{\left(x^2+1\right)^2}$ 136. $y' = 6x\cos\left(3x^2\right)$

137. $y' = \frac{6x^2 - 5}{4\left(2x^3 - 5x\right)^{\frac{3}{4}}}$ 138. $y' = e^x\left(\cos x - \sin x\right)$