

Chantilly High School

AP Calculus AB

To: All students enrolled in AP Calculus AB for the school year 2009 – 2010

From: AP Calculus AB teachers: Ms. Black and Mrs. Snyder

Going into AP calculus, there are certain skills that have been taught to you over the previous years that we assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

We assume that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example, should now be a part of you. If not, you would not be going on to AP calculus. The topics covered in the packet are skills that are used continually in AP calculus.

The attached summer packet is for all students enrolled in AP Calculus AB in the fall. This packet will be collected on the first day of school and graded. Show all work on separate paper. NO work written in the question packet will be checked.

Rather than give you a textbook to remind you of the techniques necessary to solve the problem, We have given you several websites that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, examine these websites. Don't fake your way through these problems. As stated, students are notoriously weak in them, even students who have achieved well prior to AP calculus. Use the websites.

Realize also that certain concepts are interrelated. Domain, for example, may require you to be an expert at working with inequalities. Solving quadratic equations may involve techniques used in solving fractional equations.

This packet is due the first day back in school in the fall. It will be graded. You will have a test on this material the second week of school. You need to get off to a good start so spend some quality time on this packet this summer. Do not rely on the calculator. Use the calculator only on the problems where calculator use is indicated. Half of the AP exam next year is taken without a calculator.

It is a mistake to decide to do this now. Let it go until mid-summer. We want these techniques to be relatively fresh in your mind in the fall. Also, do not wait to do them at the very last minute. These take time.

If you have any questions about any of these problems or techniques used in solving them, you may contact either of us at the school website/email address. Have a good summer and see you in the fall.

Have a great summer!

Below are listed topics in the review. You can certainly do Google searches for any of these topics. But we have given you several sites that will cover pretty much all of these topics.

Here is a good site for most algebra topics:

<http://www.purplemath.com/modules/index.htm>

Beginning algebra topics

Exponents

Negative and fractional exponents

Intermediate algebra topics

Domain

Solving inequalities: absolute value

Solving inequalities: quadratic

Special Factoring formulas

Function transformation

Factor theorem (p over q method)

Even and odd functions

Solving quadratic equations and quadratic formula

Advanced algebra topics

Asymptotes

Complex fractions

Composition of functions

Solving Rational (fractional) equations

Trig Information

<http://www.mathematicshelpcentral.com/index.html>

Once in the site, go to lecture notes.

Basic right angle trig

Trig equations

Limits

<http://www.calculus-help.com/funstuff/phobe.html>

**CHANTILLY HIGH SCHOOL
AP CALCULUS SUMMER ASSIGNMENT**

Name _____

Complete the following. Show and attach all work in a clear manner. Do NOT do any work on this sheet – all work must be separate. Have this assignment completed and ready to turn in on the first day of school.

1. Are the following statements true? If not, explain in words why not?

a) $\frac{2k}{2x+h} = \frac{k}{x+h}$

b) $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$

c) $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

d) $3\frac{a}{b} = \frac{3a}{3b}$

e) $3\frac{a}{b} = \frac{3a}{b}$

f) $3\frac{a+b}{c} = \frac{3a+b}{c}$

2. Simplify:

a) $\frac{\frac{x}{2}}{\frac{x}{4}}$

b) $h \div \frac{(x+h)}{h}$

c) $\frac{\sqrt{x-2} + \frac{5}{\sqrt{x-2}}}{x-2}$

3. Solve $xy' + y = 1 + y'$ for y'

4. Solve the equation: a) $4x^2 - 21x - 18 = 0$

b) $x^3 + 3x^2 - 5x - 15 = 0$

c) $x^4 - 9x^2 + 8 = 0$

5. Write as a single fraction with denominator in factored form: $\frac{7x^2 + 5x}{x^2 + 1} - \frac{5x}{x^2 - 6}$

6. Graph the equation $y = x^3 - x$ and answer the following questions.

- a) Is the point (3, 2) on the graph?
- b) Is the point (2, 6) on the graph?
- c) Is the function even, odd, or neither?
- d) What is the y intercept?
- e) Find the x intercepts.

7. Show work to determine if the relation is even, odd, or neither.

a) $f(x) = 2x^2 - 7$

b) $f(x) = -4x^3 - 2x$

c) $f(x) = 4x^2 - 4x + 4$

8. Find the equation of the straight line that passes through the point (2, 4) and is parallel to the line $2x + 3y - 8 = 0$.

9. Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point (1,2)

10. The line with the slope 5 that passes through the point (-1,3) intersects the x axis at a point. What are the coordinates of this point?

11. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2,4) intersects the y axis?

12. Given $f(x) = |x-3| - 5$ find $f(1) - f(5)$.

13. Given $f(x) = x^2 - 3x + 4$ find $f(x+2) - f(2)$.

14. Find the domain for each of the following functions.

a) $h(x) = \frac{1}{4x^2 - 21x - 18}$

b) $k(x) = \sqrt{x^2 - 5x - 14}$

c) $p(x) = \frac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$

d) $y = \ln(2x - 12)$

15. Find $f(x + \Delta x)$ for $f(x) = x^2 - 2x - 3$.

16. Find $\frac{f(x + \Delta x) - f(x)}{\Delta x}$ if $f(x) = 8x^2 + 1$

17. Given $f(x) = \frac{1}{x}$ Find: $\frac{f(x + h) - f(x)}{h}$

18. Sketch the graph of each function

a) $f(x) = \begin{cases} 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$

b) $f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x + 3 & (2, \infty) \end{cases}$

19. Given $f(x) = x - 3$ and $g(x) = \sqrt{x}$ complete the following

a) $f(g(x)) =$ b) $g(f(x)) =$ c) $f(f(x)) =$

20. Given $f(x) = \frac{1}{x-5}$ and $g(x) = x^2 - 5$ complete the following

a) $f(g(7)) =$ b) $g(f(v)) =$ c) $g(g(x)) =$

21. Let $f(x) = 2x - 2$. Complete the following:

- a) Sketch the graph of $f(x)$.
- b) Determine whether f has an inverse function.
- c) Sketch the graph of $f^{-1}(x)$
- d) Give the equation for $f^{-1}(x)$

22. Simplify using only positive exponents. Do not rationalize the denominator.

a) $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ b) $\left(\frac{1}{x^2} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^2}\right)^{-\frac{1}{2}}$

23. If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$.

- a) $f(x) - 4$ b) $f(x - 4)$ c) $-f(x + 2)$
- d) $5f(x) + 3$ e) $f(2x)$ f) $|f(x)|$

24. Find the surface area of a box of height h whose base dimensions are p and q , and that satisfies the following condition:

- a) The box is closed.
- b) The box has an open top.
- c) The box has an open top and a square base with side length p .

25. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression (in terms of x) for the distance from the foot of the ladder to the base of the wall.

26. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x .

27. Evaluate: Answers for g & h must be in radians.

a) $\cos 0$ b) $\sin 0$ c) $\tan \frac{\pi}{2}$ d) $\cos \frac{\pi}{4}$ e) $\sin \frac{\pi}{2}$
f) $\sin \pi$ g) $\arccos \frac{\sqrt{3}}{2}$ h) $\arctan 1$

28-30. Find the solution of the equations for $0 \leq x < 2\pi$

28. $2 \sin^2 \theta = 1 - \sin \theta$ 29. $2 \tan \theta - \sec^2 \theta = 0$ 30. $\sin 2\theta + \sin \theta = 0$

31. Which of the following expressions are identical?

a) $\cos^2 x$ b) $(\cos x)^2$ c) $\cos x^2$

32. Which of the following expressions are identical?

a) $(\sin x)^{-1}$ b) $\arcsin x$ c) $\sin x^{-1}$ d) $\frac{1}{\sin x}$

33. Solve for x .

a) $\ln e^3 = x$ b) $\ln e^x = 4$ c) $\ln x + \ln x = 0$
d) $e^{\ln 5} = x$ e) $\ln 1 - \ln e = x$ f) $\ln 6 + \ln x - \ln 2 = 3$
g) $\ln(x+5) = \ln(x-1) - \ln(x+1)$

34-41: Evaluate the limit.

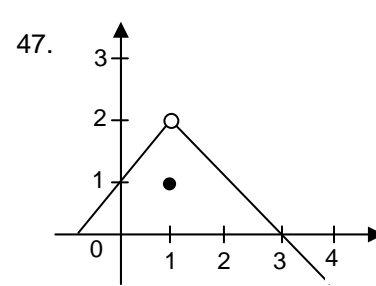
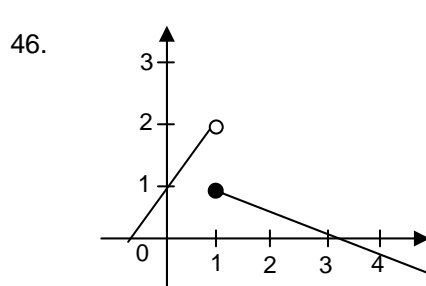
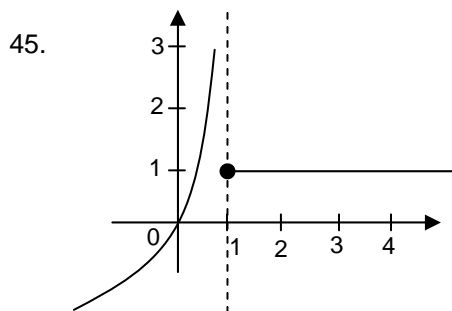
34. $\lim_{x \rightarrow 3} (x^2 + 2)$ 35. $\lim_{x \rightarrow -3} \frac{(x+3)(x-4)}{(x+3)(x+1)}$ 36. $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$
37. $\lim_{x \rightarrow -2} \frac{x-4}{x^2 - 2x - 8}$ 38. $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x^2 + 7x + 12}$ 39. $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$
40. $\lim_{x \rightarrow 5} \frac{x-5}{|x-5|}$ 41. $\lim_{x \rightarrow 8} \frac{1}{x-8}$

42-47 For each of the following determine: a) $\lim_{x \rightarrow 1^-} f(x)$ b) $\lim_{x \rightarrow 1^+} f(x)$ and c) $\lim_{x \rightarrow 1} f(x)$

$$42. f(x) = \begin{cases} x^2 - 1 & x < 1 \\ 4 - x & x \geq 1 \end{cases}$$

$$43. f(x) = \begin{cases} 3x - 1 & x \leq 1 \\ 3 - x & x > 1 \end{cases}$$

$$44. f(x) = \begin{cases} -x^2 & x < 1 \\ 2 & x = 1 \\ x - 2 & x > 1 \end{cases}$$



Answers

1 a. No, can not cancel terms.

1 b. No, can not split the denominator

1 c. Yes

1 d. No, 3 is multiplied by fraction not top and bottom

1 e. Yes

1 f. No, must distribute 3 to every term

2 a. 2

2 b. $\frac{h^2}{x+h}$

2 c. $\frac{x+3}{(x-2)^{\frac{3}{2}}}$

3. $y' = \frac{1-y}{x-1}$

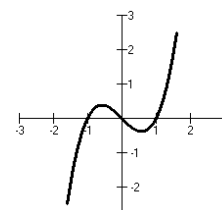
4 a. $x = 6, x = -\frac{3}{4}$

4 b. $x = -3, x = \pm\sqrt{5}$

4 c. $x = \pm 1, x = \pm 2\sqrt{2}$

5. $\frac{7x(x^3 - 6x - 5)}{(x^2 + 1)(x^2 - 6)}$

6.



6 a. No

6 b. Yes

6 c. Odd

6 d. (0, 0)

6 e. (-1, 0) (0, 0) (1, 0)

7 a. even

7 b. odd

7 c. neither

8. $y - 4 = -\frac{2}{3}(x - 2)$

9. $y - 2 = \frac{3}{2}(x - 1)$

10. $\left(-\frac{8}{5}, 0\right)$

11. $\left(0, -\frac{2}{3}\right)$

12. 0

13. $x^2 + x$

14 a. $\left(-\infty, -\frac{3}{4}\right) \cup \left(-\frac{3}{4}, 6\right) \cup (6, \infty)$

14 b. $(-\infty, -2] \cup [7, \infty)$

14 c. $(-\infty, -5) \cup (6, \infty)$

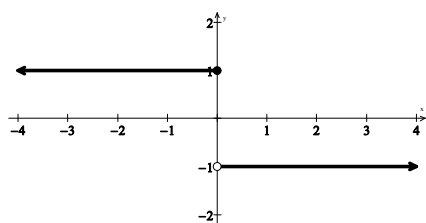
14 d. $(6, \infty)$

15. $x^2 + 2x(\Delta x) + (\Delta x)^2 - 2x - 2(\Delta x) - 3$

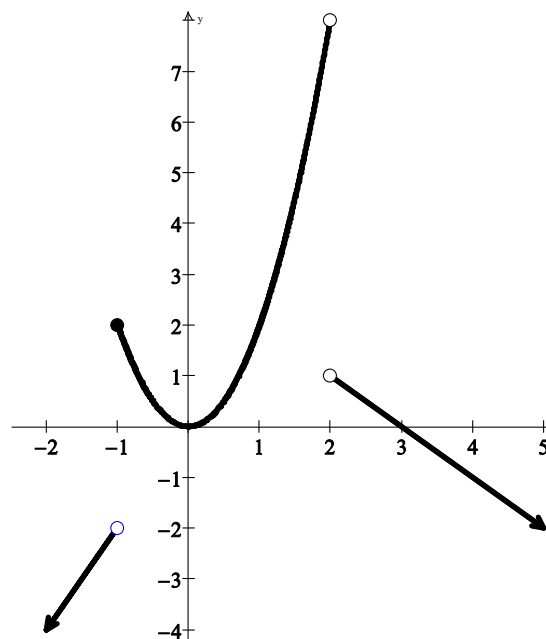
16. $16x + 8(\Delta x)$

17. $\frac{-1}{x(x+h)}$

18 a.



18 b.



19 a. $\sqrt{x} - 3$

19 b. $\sqrt{x-3}$

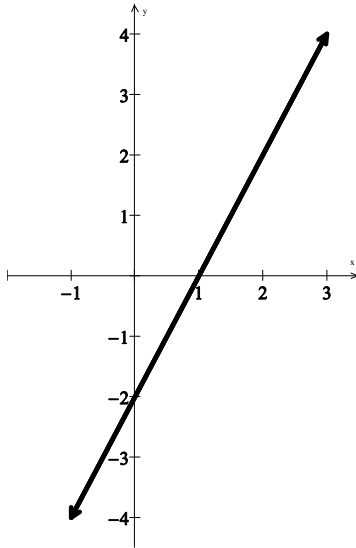
19 c. $x - 6$

20 a. $\frac{1}{39}$

20 b. $\frac{-5v^2 + 50v - 124}{(v-5)^2}$

20 c. $x^4 - 10x^2 + 20$

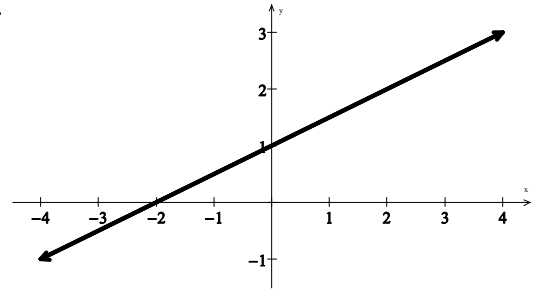
21 a.



21 b. Yes

21 c.

21 d. $y = \frac{1}{2}x + 1$



22 a. $\frac{2}{(x-4)^{\frac{1}{4}}}$

22 b. $\frac{1}{\sqrt{x^2 + 4xy + y^2}}$

23 a. down 4 units

23 b. right 4 units

23 c. left 2 units, reflect over x-axis

23 d. up 3 units, vertical stretch by 5

23 e. horizontal compression by $\frac{1}{2}$

23 f. reflects all negative y values over the x-axis

24 a. $2pq + 2(p+q)h$

24 b. $pq + 2(p+q)h$

24 c. $p^2 + 4ph$

25. $\sqrt{7^2 - x^2}$

26. $\frac{x^2}{16} + \frac{(5-x)^2}{4\pi}$

27 a. 1

27 b. 0

27 c. undefined

27 d. $\frac{1}{\sqrt{2}}$

27 e. 1

27 f. 0

27 g. $\frac{\pi}{6}$

27 h. $\frac{\pi}{4}$

28. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

29. $\frac{\pi}{4}, \frac{5\pi}{4}$

30. $0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

31. a & b

32. a & d

33 a. 3

33 b. 4

33 c. 1

33 d. 5

33 e. -1

33 f. $\frac{1}{3}e^3$

33 g. no solution

34. 11

35. $\frac{7}{2}$

36. $\frac{1}{10}$

37. DNE

38. -4

39. 12

40. DNE

41. DNE

42 a. 0

42 b. 3

42 c. DNE

43 a. 2

43 b. 2

43 c. 2

44 a. -1

44 b. -1

44 c. -1

45 a. ∞

45 b. 1

45 c. DNE

46 a. 2

46 b. 1

46 c. DNE

47 a. 2

47 b. 2

47 c. 2